Introductory Statistics
8: Risk Differences

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https://youtu.be/TDq9JpHCdXg

Lecture Video at above link

Summary

Very short lecture:

How do determine whether two proportions (percentages, risk levels) are statistically significant?

→ We already know from last class how to compare **ratio** of proportions (“risk ratio”)

→ What if we want to directly compare ***difference***?

Ratio vs. Difference of Proportions

	CHD	No CHD	SUM
Estrogen/Progestin	164	8342	8506
Placebo	122	7980	8102
SUM	286	16322	16608

When we started did you think: *Isn't there an easier way to do this?*

164/8506 = 1.93% (percentage of hormone group who got CHD)
122/8102 = 1.51% (percentage of placebo group who got CHD)

Obviously, 1.93% is bigger than 1.51%... right?

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Obviously, 1.93% is bigger than 1.51%... right?

1.93% - 1.51% = 0.42%

→ If we directly compare these, we are testing the ***difference of two proportions*** (Risk Difference).

(previously we did the ***ratio***: 1.93% / 1.51% = 1.28x)

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Obviously, 1.93% is bigger than 1.51%… right?

Here is the dangerous part:
1.93% – 1.51% = 0.42%
Should we say +0.42% chance of getting CHD in hormone group?

Better example: Semmelweis data

Ignaz Semmelweis (1818-1865)

Assistant physician at the Vienna General Hospital Maternity Clinic from 1846-1849

Investigated Child-bed fever

Ignaz Philipp Semmelweis (1818–65).

	1st ward: Physicians			2nd ward: Midwifes		
	births	dead	%	births	dead	%
1841	3036	237	7·7	2442	86	3·5
1842	3287	518	15·8	2659	202	7·5
1843	3060	274	8·9	2739	164	5·9
1844	3157	260	8·2	2956	68	2·3
1845	3492	241	6·8	3241	66	2·03
1846	4010	459	11·4	3754	105	2·7
Summa	20042	1989	9·92	17791	691	3·38

Semmelweis, 1861

Semmelweis’s data

Percentage of mothers who died in childbed:

1st ward, Physicians: 9.92%   (n=20042)

2nd ward, Midwifes: 3.88%   (n=17791)

“Reviewer’s” response:

Just a fluke!

(no joke)

Semmelweis's data

Percentage of mothers who died in childbed:

1st ward, Physicians: 9.92% (n=20042)

2nd ward, Midwifes: 3.88% (n=17791)

We could use risk ratio… 9.92% / 3.88% = 2.56 x

But instead we'll try comparing "directly"
9.92% - 3.88% = 6.04% *higher* death rate in ward 1

→ How can we check if this is "real" or not?
1) Hypothesis test
2) Confidence intervals

Binomial Distribution

We need to remember where our proportions come from: *Binomial Distribution*

Distribution of “number of successes” *X* out of *N* trials with probability of success *p*

Binomial Distribution

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Distribution of “number of successes” *X* out of *N* trials with probability of success *p*

Probability of *failure* is (1-p)

→ We call this “*q*”

q=(1-p)

Coins

How can we determine whether this result was random or not?

Let's start with a simple model: the coin…

Head

Tail

P("Head") = p = 0.5

P("Tail") = 1 - p = q = 0.5

Coin

Now, we repeatedly flip a coin and count the number of Heads.

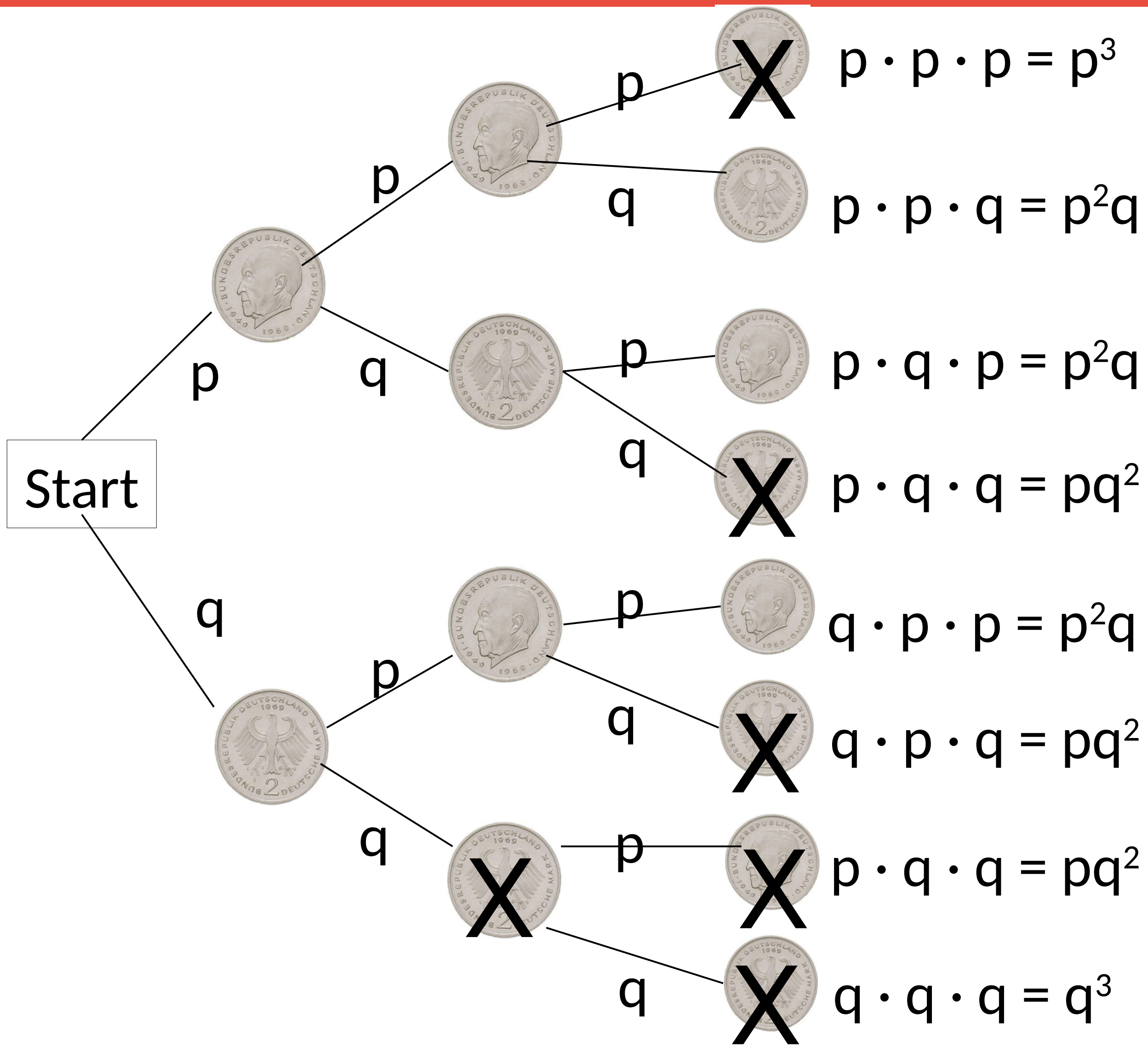
Let's say, 3 times:

X = Number of successes (Heads) = 2

What is the probability for this result?

P(X=2) = ?

Coin


Start
p
q
p
q
p
q
p
q
p
q
p
q
p
q
p · p · p = p3
p · p · q = p2q
p · q · p = p2q
p · q · q = pq2
q · p · p = p2q
q · p · q = pq2
p · q · q = pq2
q · q · q = q3

Coin

X = Number of successes (Heads) = 2

What is the probability for this result?

$$P(X=2) = 3 \cdot p^2 q = 3 \cdot 0.5 \cdot 0.5 \cdot 0.5 =$$

$$3 \cdot 0.125 = 0.375$$

$$P(X=x) = \textcolor{red}{?} \cdot p^x q^{n-x}$$

Coin

Let’s toss the coin 5 times:

X = Number of successes (Heads) = 2

What is the probability for this result?

$P(X=2) = \textcolor{red}{?} \cdot p^2 q^{(5-2)}$

How many possibilities to arrange 2 Heads in a sequence of 5 coin flips….

Coin: Binomial

How many possibilities to select 2 positions in a sequence of 5?

“n choose x”:

$$\binom{n}{x} = \frac{n!}{x!(n-x)!}$$

in our example:

$$\binom{5}{2} = \frac{5!}{2!(5-2)!} = \frac{5 \times 4}{2} = 10$$

$$P(X=2) = \binom{5}{2} p^2 q^3 = 10 \times 0.5^2 \times 0.5^3 = 0.3125$$

Binomial: General Formula

$$P(X=x)=\binom{n}{x}p^{x}q^{(n-x)}$$

$$\binom{n}{x}=\frac{n!}{x!(n-x)!}$$

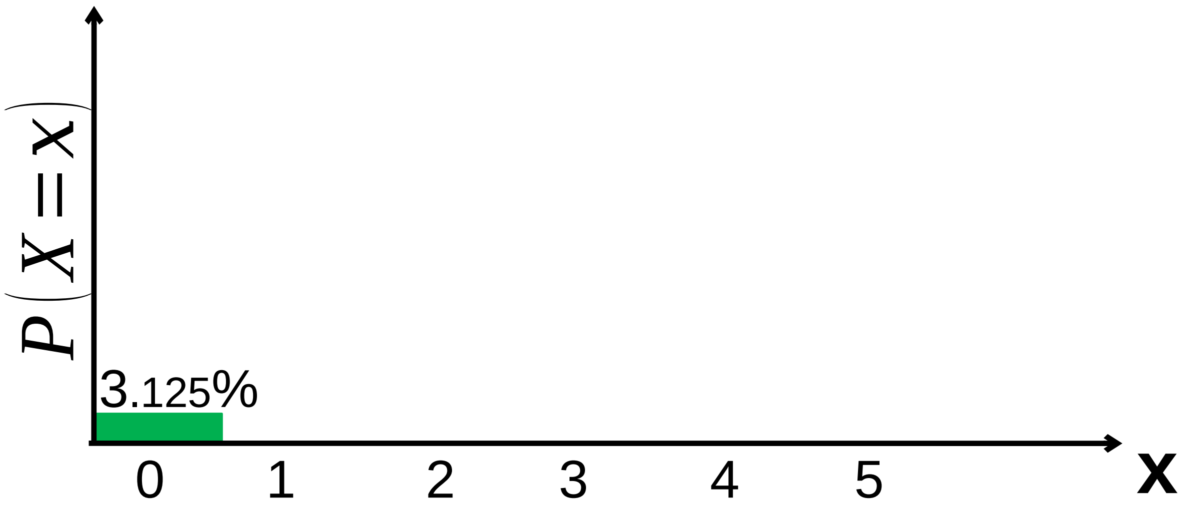
P(X=x) is the probability to have x successes in n number of trials.

p is the probability for success in a single trial
q = 1 - p

Binomial for coin (p=0.5, n=5)

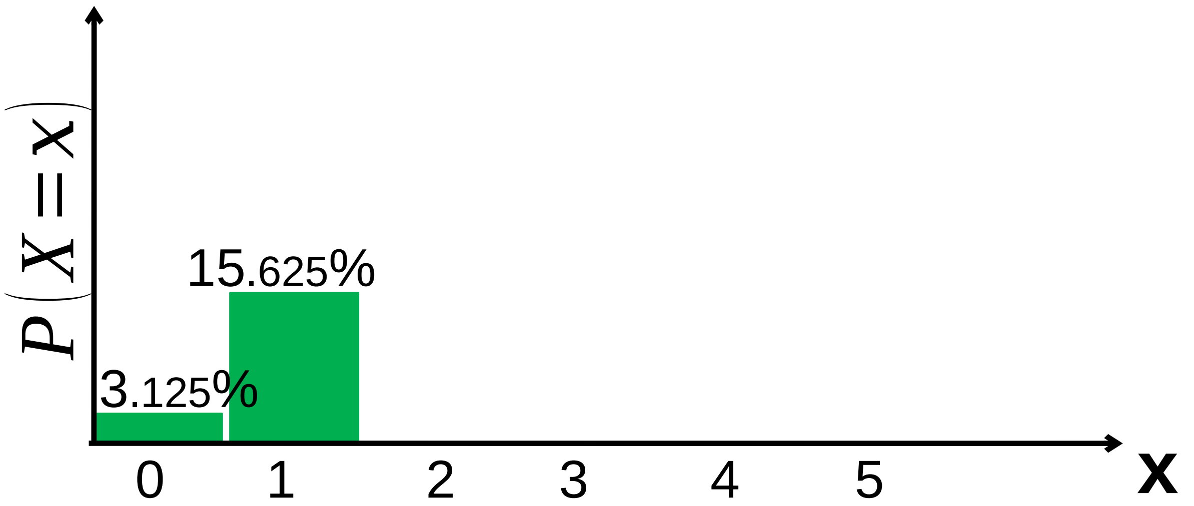
Distribution for 5 coin flips:

$$P(X=0)=\binom{5}{0}0.5^{0}0.5^{5}=1\times 0.03125=0.03125$$



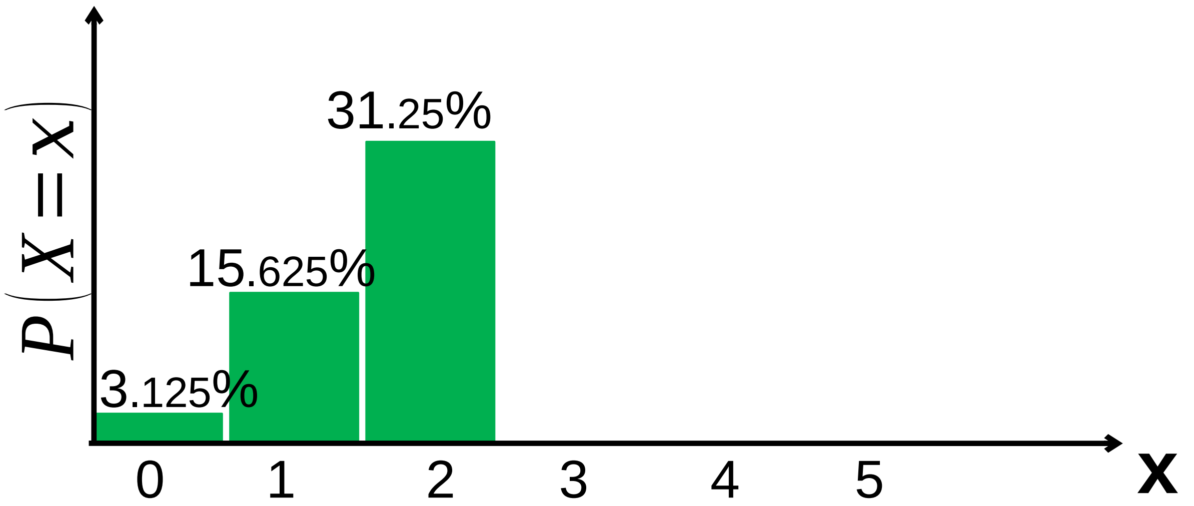
Binomial for coin (p=0.5, n=5)

$$P(X=1)=\binom{5}{1}0.5^{1}0.5^{4}=5\times 0.03125=0.15625$$



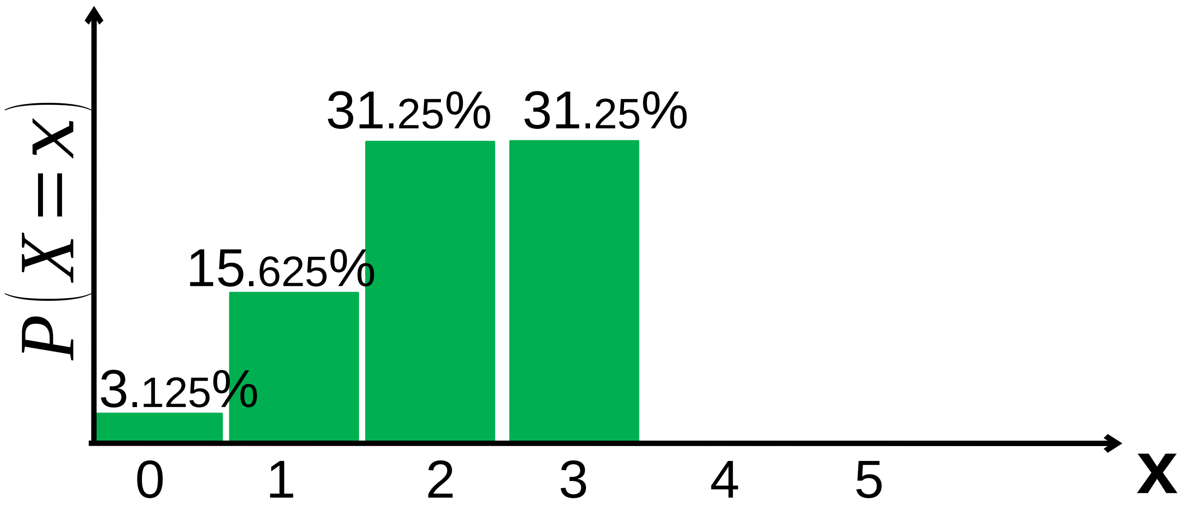
Binomial for coin (p=0.5, n=5)

$$P(X=2)=\binom{5}{2}0.5^2\,0.5^3=10\times 0.03125=0.31250$$



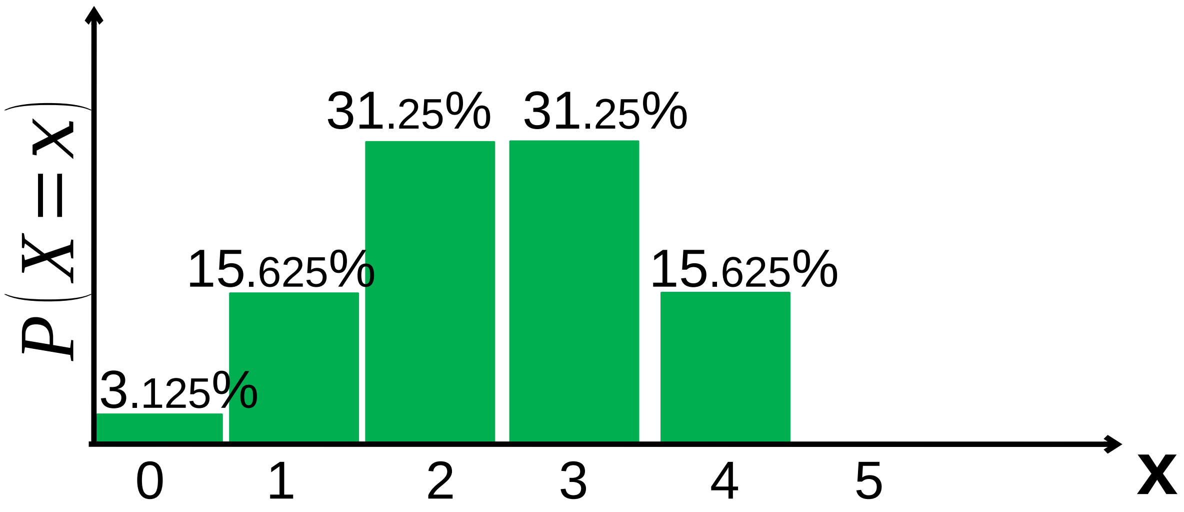
Binomial for coin (p=0.5, n=5)

$$P(X=3)=\binom{5}{3}0.5^{3}0.5^{2}=10\times 0.03125=0.31250$$



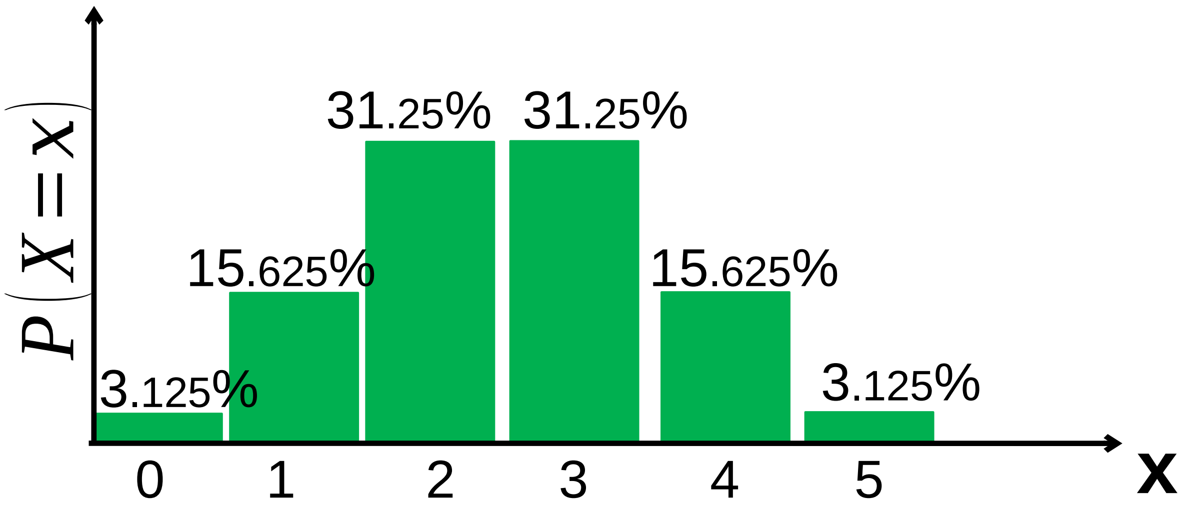
Binomial for coin (p=0.5, n=5)

$$P(X=4)=\binom{5}{4}0.5^4\,0.5^1=5\times0.03125=0.15625$$



Binomial for coin (p=0.5, n=5)

$$P(X=5)=\binom{5}{5}0.5^{5}0.5^{0}=1\times 0.03125=0.03125$$



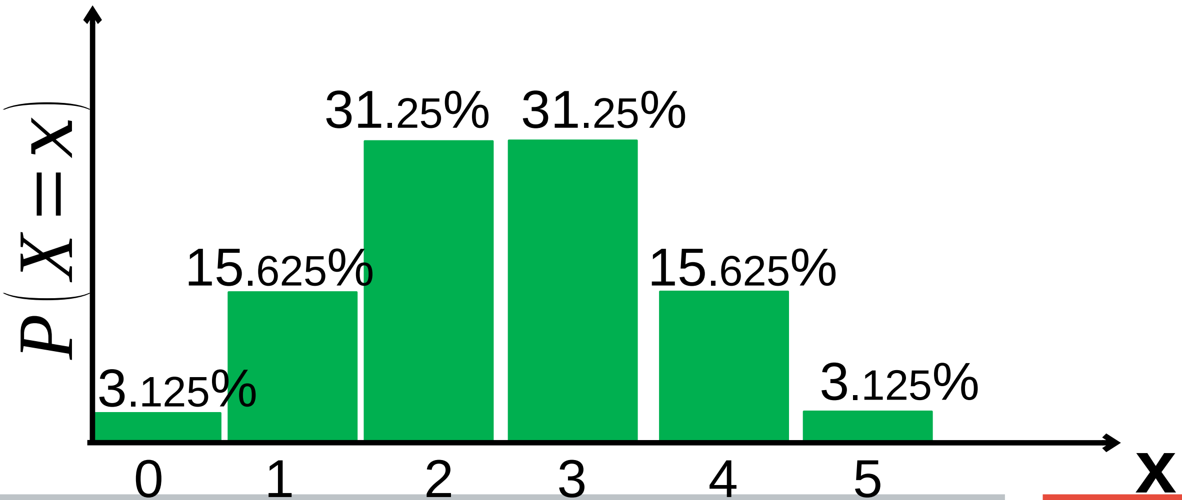
Hypothesis test with Binomial

Let's assume we want to test whether a coin is "biased":

Null Hypothesis (H_0): $p_1 = 0.5$

Alternative Hypothesis (H_a): $p_1 \neq 0.5$

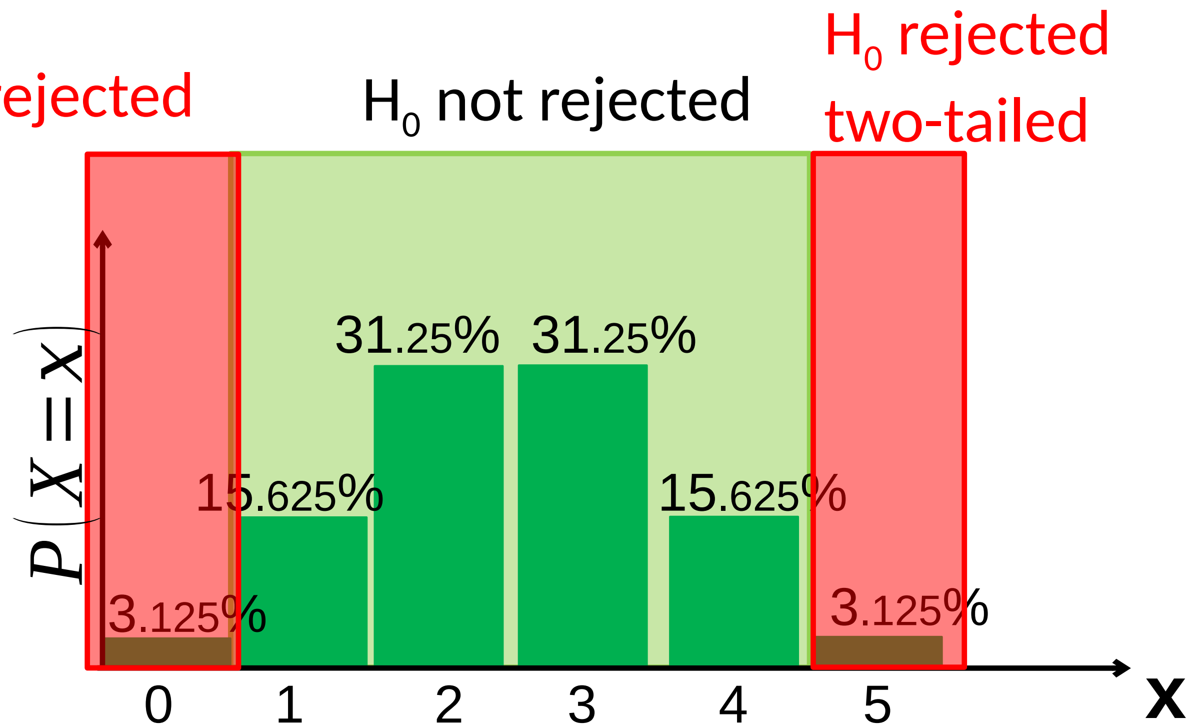
Significance level α = 0.1 two-tailed test!



Hypothesis test with binomial

We reject the null hypothesis if 5 coin flips reveal a value of “0” or “5”:

because P(X=0)+P(X=5) ≤ α ; α = 0.1

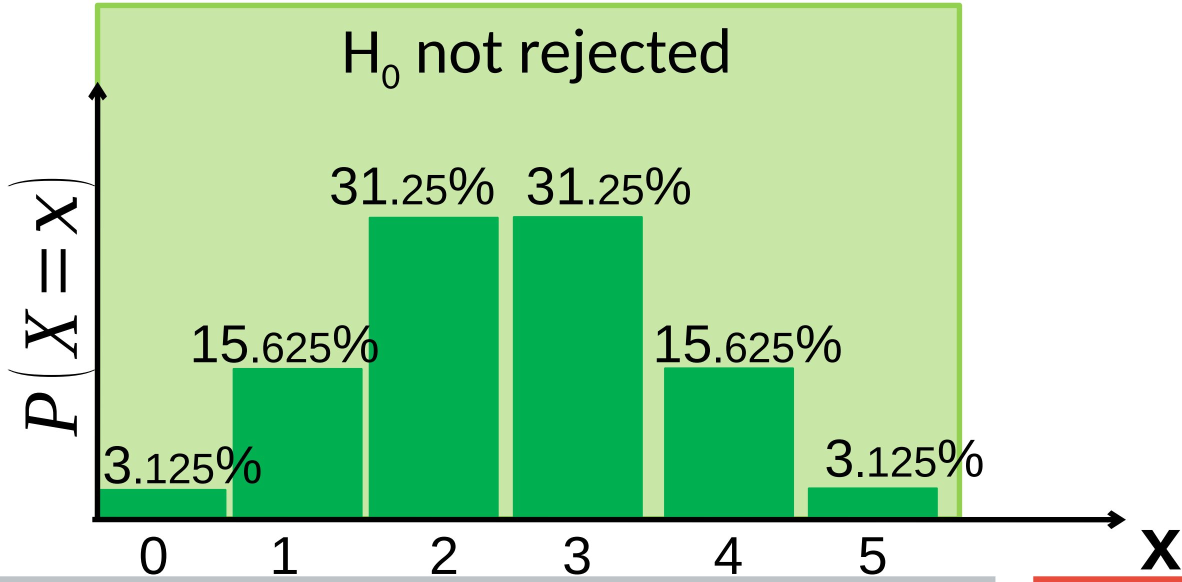


Hypothesis test with binomial

We cannot reject the null hypothesis (two-tailed, **α=0.05**) with 5 coin flips.

because P(X=0)+P(X=5) = 6.25 > α ; **α = 0.05**

No possibility to reject H_0 in a two-tailed test -> small sample size!



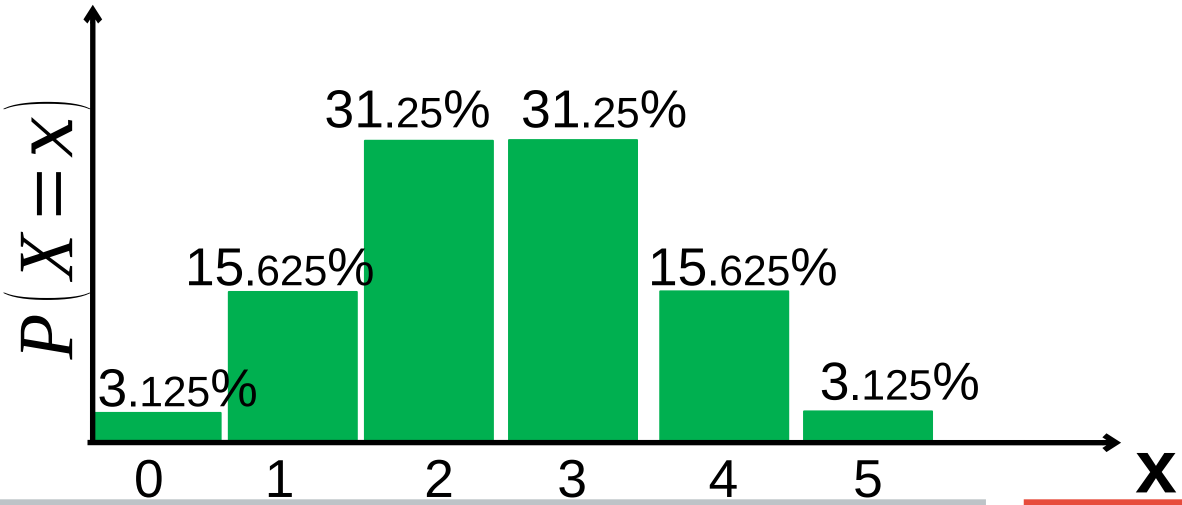
One-tailed hypothesis test…

Let's assume we want to test whether a coin is "biased" toward "Heads":

Null Hypothesis (H_0): $p_1 \leq 0.5$

Alternative Hypothesis (H_a): $p_1 > 0.5$

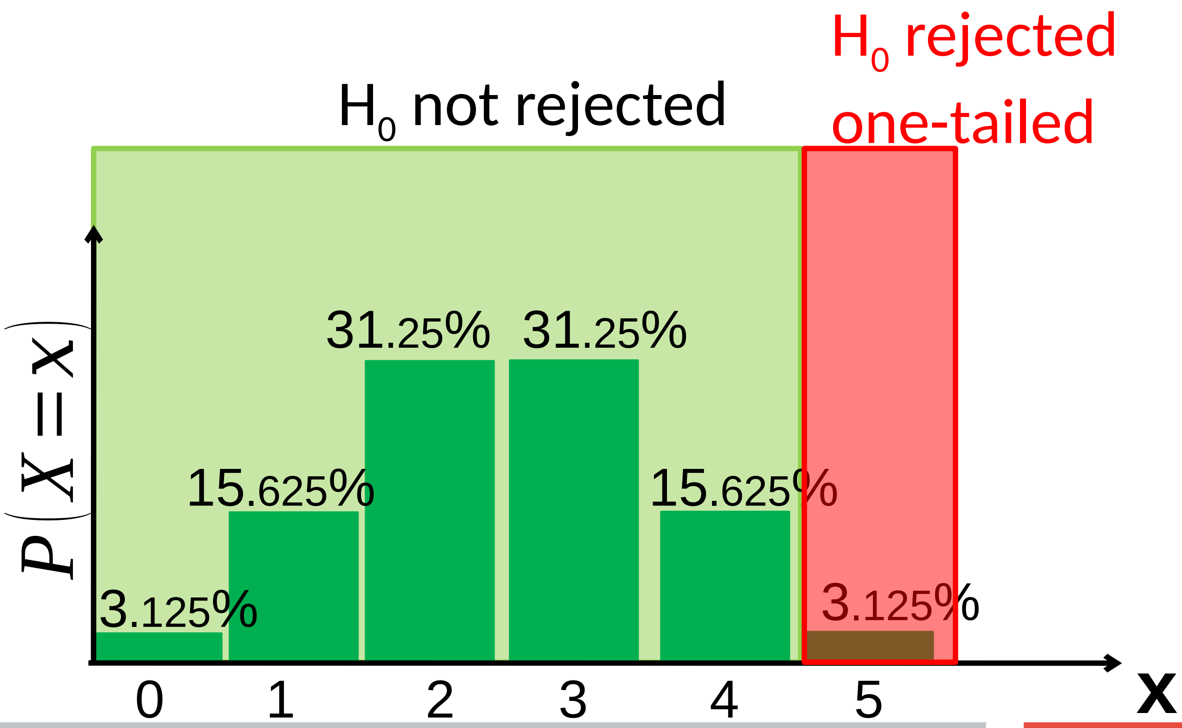
Significance level α = 0.05 **one-tailed test**



One-tailed hypothesis test

We reject the null hypothesis if 5 coin flips reveal a value of or “5”.

because P(X=5) =3.125 ≤ α ; α = 0.05



Test of one proportion (exact)

This is like Fisher's exact test – we're testing whether *p* is different than a *target value*

We exactly calculate the probabilities for the full binomial distribution with our sample numbers and probability, and can compute the cut-off.

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→ This gets very computationally complex as *n* gets large...but no problem for modern computers.

Test of one proportion (exact)

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What about if we have *two* observed proportions p_1 and p_2? Can we check if p_1 is different than p_2?

Difference of two coins

Now, let's assume we have two coins, and they seem to be different…

Null Hypothesis (H_0): $p_1 = p_2$

Alternative Hypothesis (H_a): $p_1 \neq p_2$

Significance level $\alpha = 0.1$ **two-tailed test**

p1: probability to get "Head" for a single coin flip with coin1
p2: probability to get "Head" for a single coin flip with coin2

X1: Number of "Heads" with coin1
X2: Number of "Heads" with coin2

Difference of two coins

Null Hypothesis (H_0): $p_1 = p_2$

Alternative Hypothesis (H_a): $p_1 \neq p_2$

Significance level $\alpha = 0.1$ **two-tailed test**

We will flip coin1 five times and coin2 five times and compute X1-X2 to test if $p_1 - p_2 = 0$

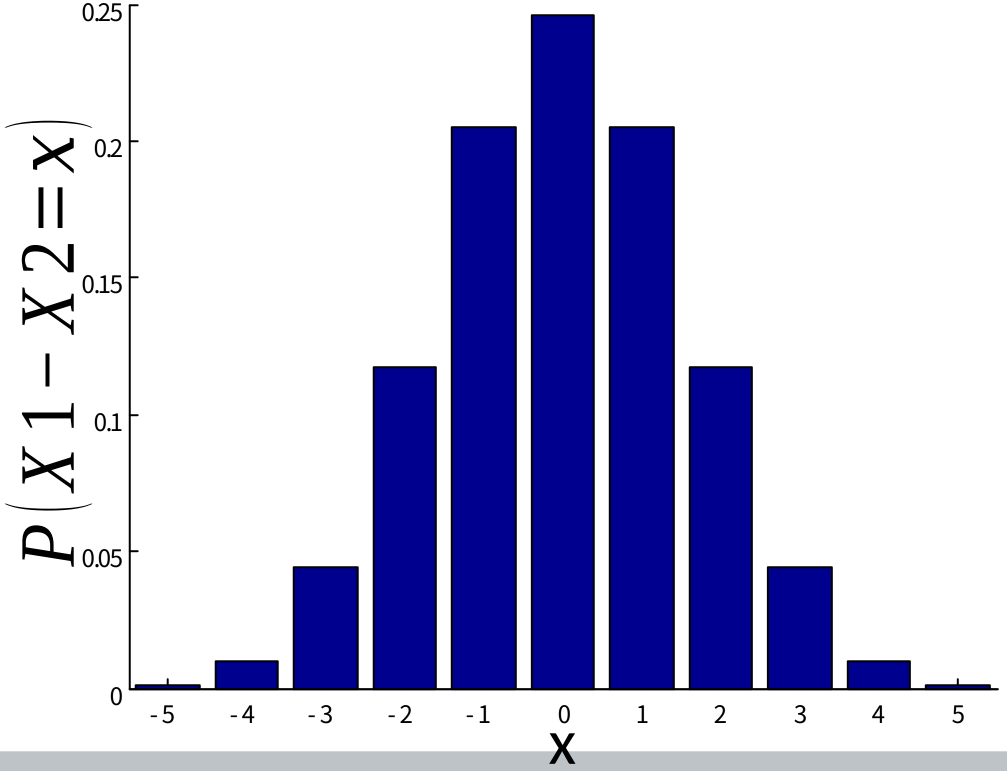
How can we derive the probability distribution for X1-X2?

Possibilities for X1-X2 (5 flips): -5, -4, -3, -2, -1, 0, +1, +2, +3, +4, +5

Difference of two coins

Probability distribution for X1-X2?

P(X1-X2=-5) = P(X1=0) · P(X2=5) = 0.03125 · 0.03125 ~= 0.00098
P(X1-X2=-4) = P(X1=0) · P(X2=4) + P(X1=1) · P(X2=5) ~= 0.00976
Etc…



Exact hypothesis test (difference coins)

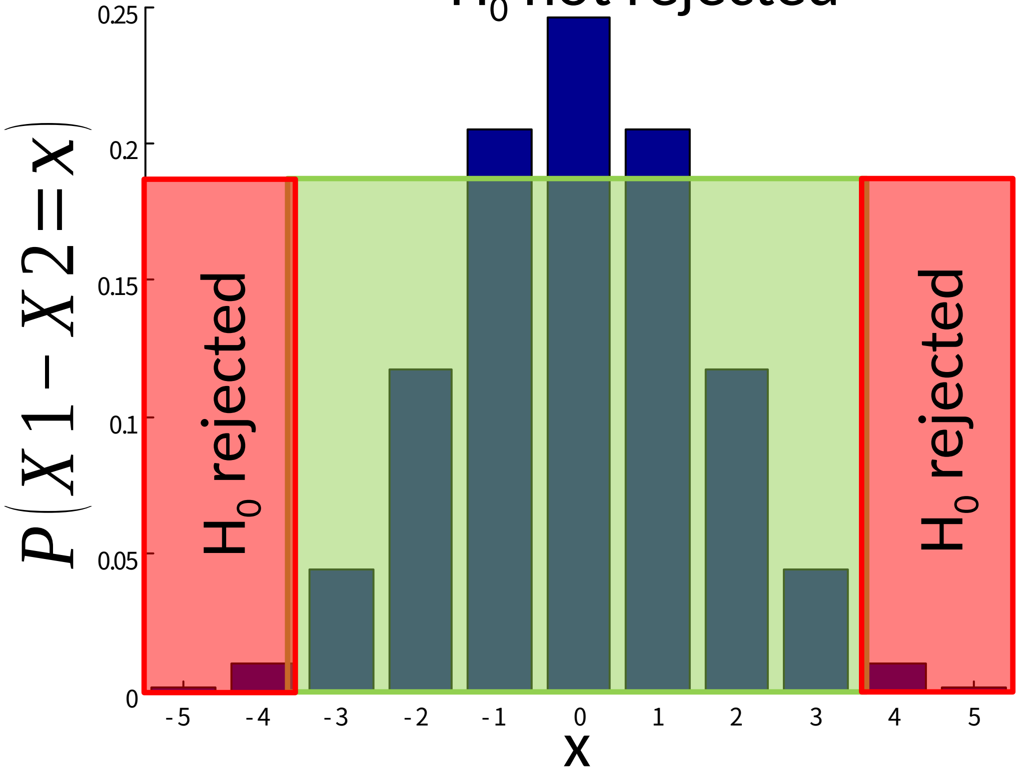
Null Hypothesis (H_0): $p_1 = p_2$

Alternative Hypothesis (H_a): $p_1 \neq p_2$

Significance level $\alpha = 0.05$ **two-tailed test**

$P(X \leq -4) + P(X \geq 4) \approx 0.021 \leq \alpha$

H_0 not rejected



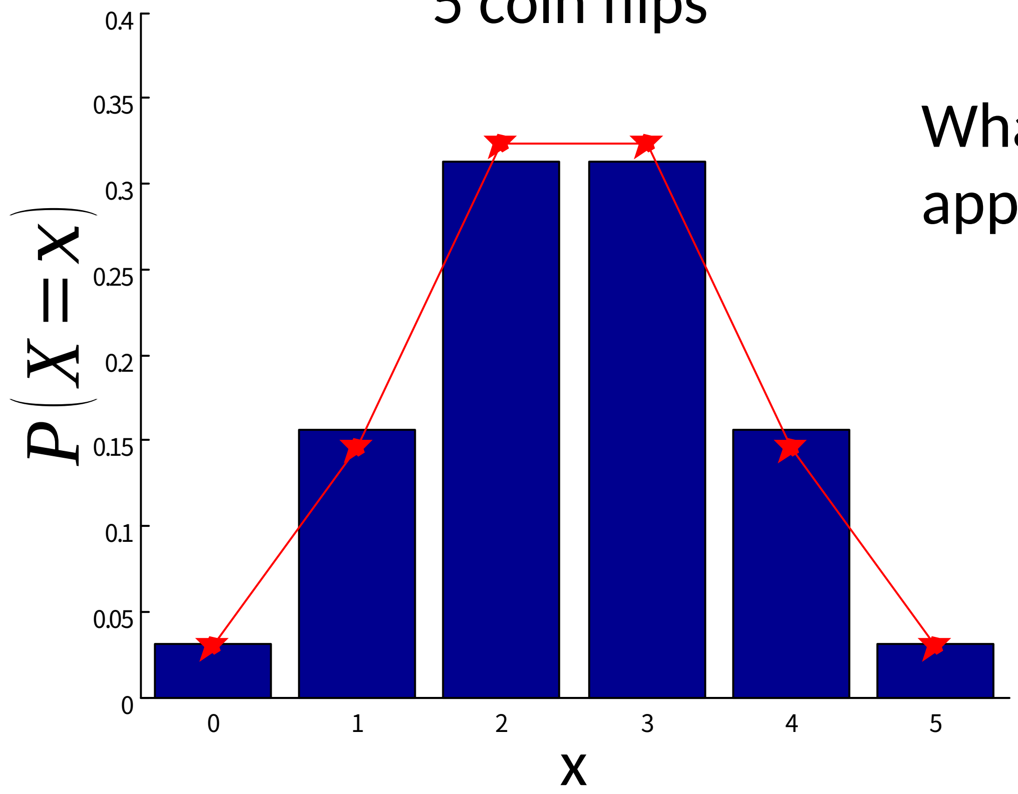
Bigger Samples...

This gets computationally complex for large samples.

Approximate as normal…

That's really tedious, what can we do for large samples?
e.g., Semmelweis: N1=20042, N2=17791

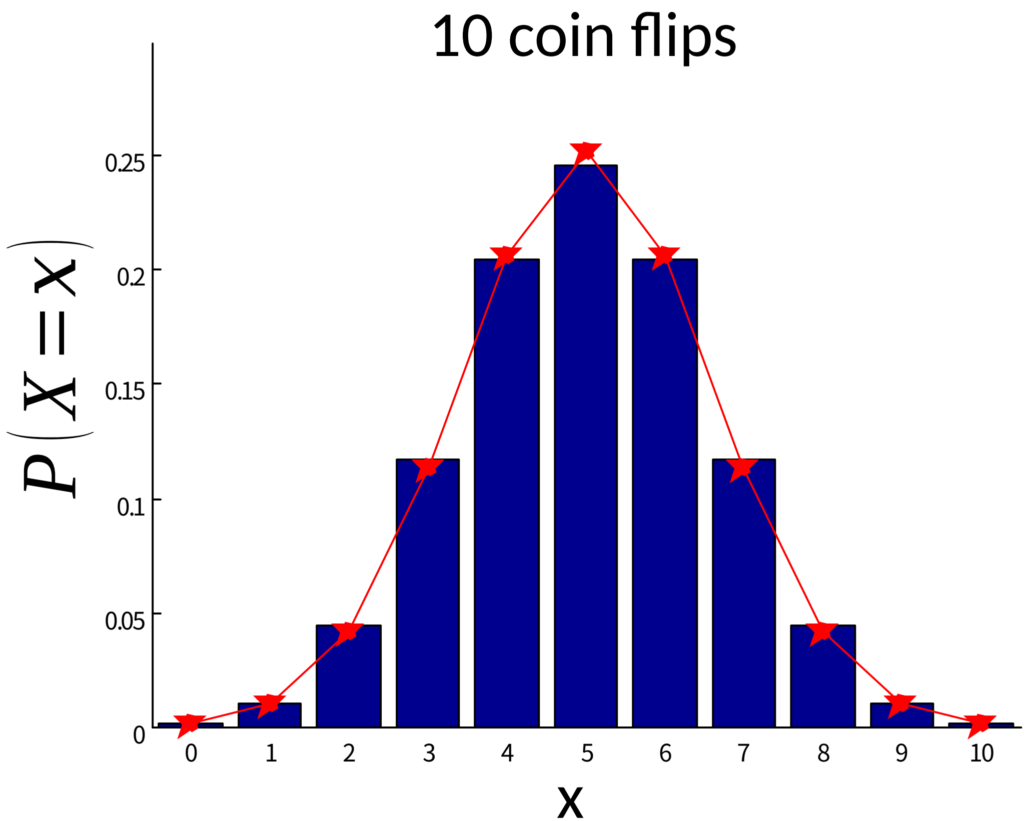
5 coin flips



What could the red approximating stars be?

Approximate as normal...

We can approximate the binomial distribution with a normal distribution: $N(\mu,\sigma^2) = N(np,npq)$



Expected value:

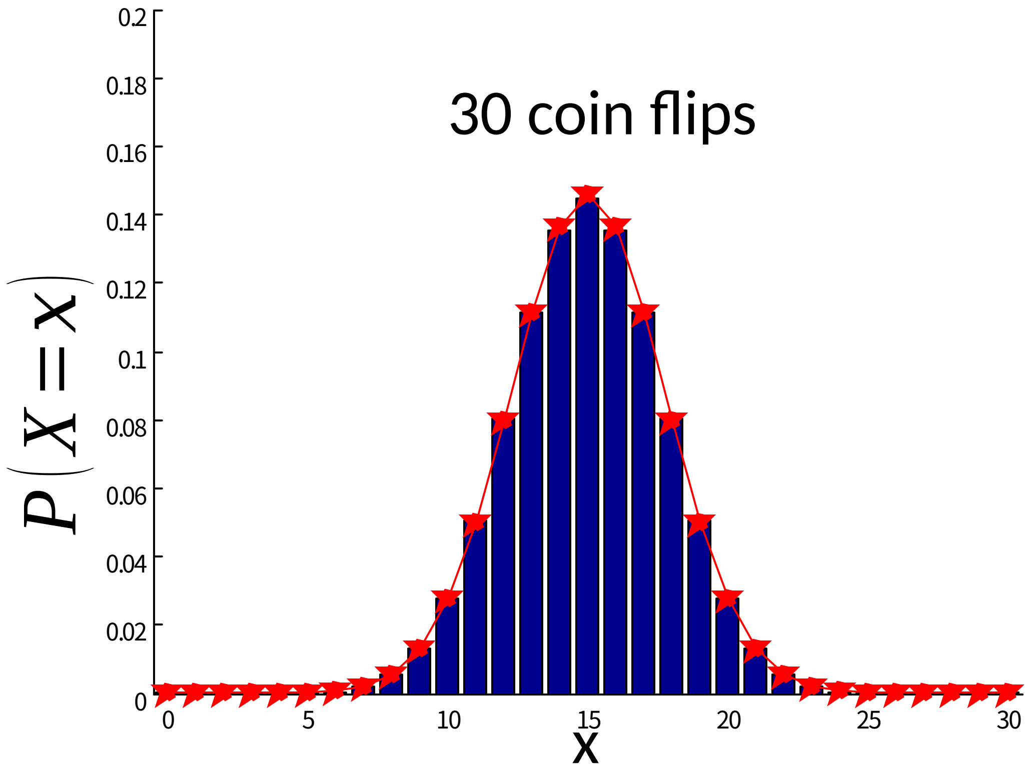
$E(X) = np \quad = 5$

$$SD(X) = \sqrt{npq} = 1.58$$

(SD: Standard deviation)

Approximate as normal...

X~N(np,npq)



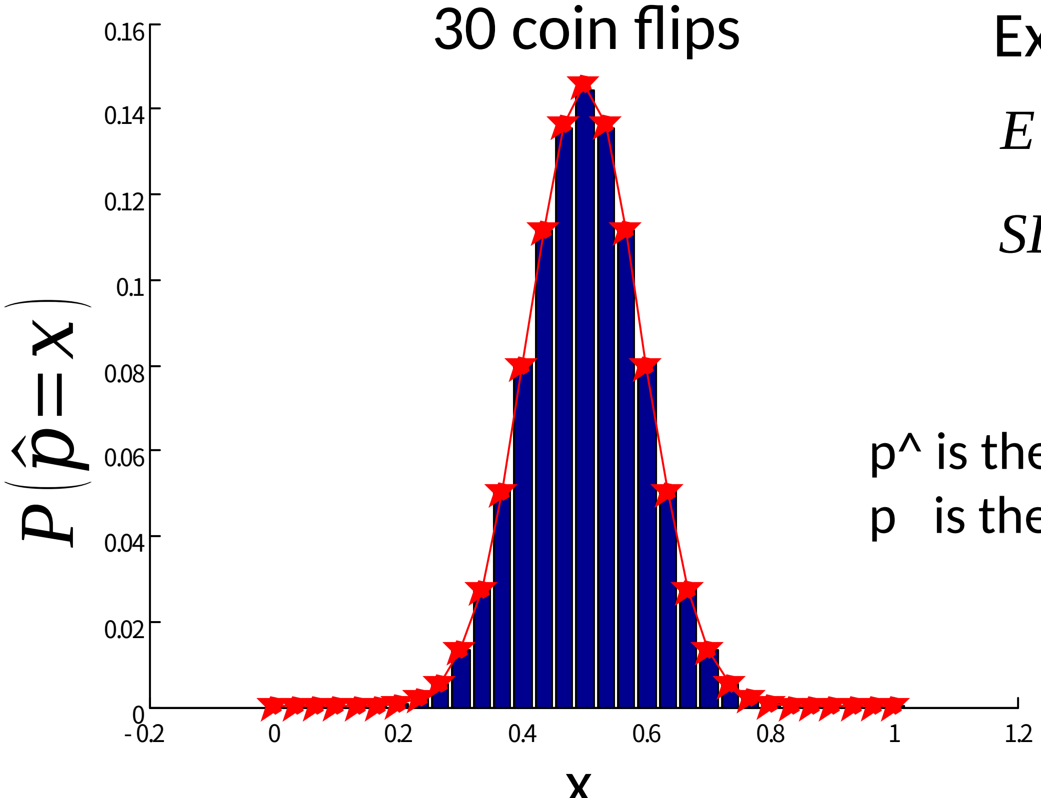
Expected value:

E(X) = np = 15

$$SD(X) = \sqrt{npq} = 2.74$$

Sample distribution

We can also provide the distribution for the estimated proportion (the little hat means estimation from a sample) rather than X.
Approximation with a normal distribution N(p,pq/n).



Expected value:

$$E(\hat{p})=p=0.5$$

$$SD(\hat{p})=\sqrt{pq/n}=0.091$$

p^ is the estimated probability
p is the true probability that is estimated

Sample distribution

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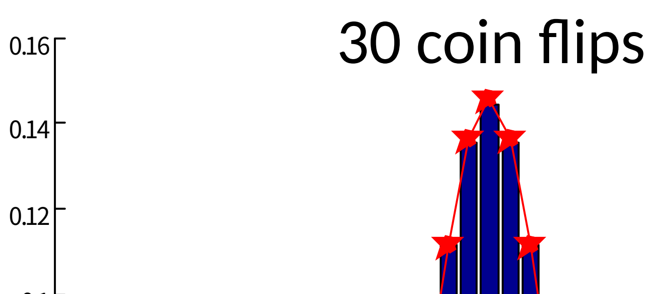


Expected value:

$$E(\hat{p}) = p = 0.5$$

$$SD(\hat{p}) = \sqrt{pq/n} = 0.091$$

Very Important:

We don't know what *p* is.
p is the true value we are trying to guess.

But we can use *delta method* etc. to estimate population parameters from sample parameters...

Mean/Var of Binomial Distribution

How do we know mean of binomial is *np* and variance is *npq*?

It's derived from the equation for Binomial:

$$P(X=x)=\binom{n}{x}p^x q^{(n-x)}$$

→ Here is a good derivation:
http://scipp.ucsc.edu/~haber/ph116C/NormalApprox.pdf

Normalize to compare in "standard Z space" (mean 0 and variance 1)

To "normalize" we want to divide by the standard deviation.

Maximum likelihood estimator (MLE) is *np*

Expected SD is sqrt(npq)

$$\frac{np}{\sqrt{npq}} \rightarrow \frac{p}{\sqrt{npq/n}} \rightarrow \frac{p^2}{npq/n^2} \rightarrow \frac{p^2}{pq/n} \rightarrow \frac{p}{\sqrt{pq/n}}$$

And we want to center our measured p hat at p0, our target value.
→ Normalize both by the standard deviation of our "target" distribution

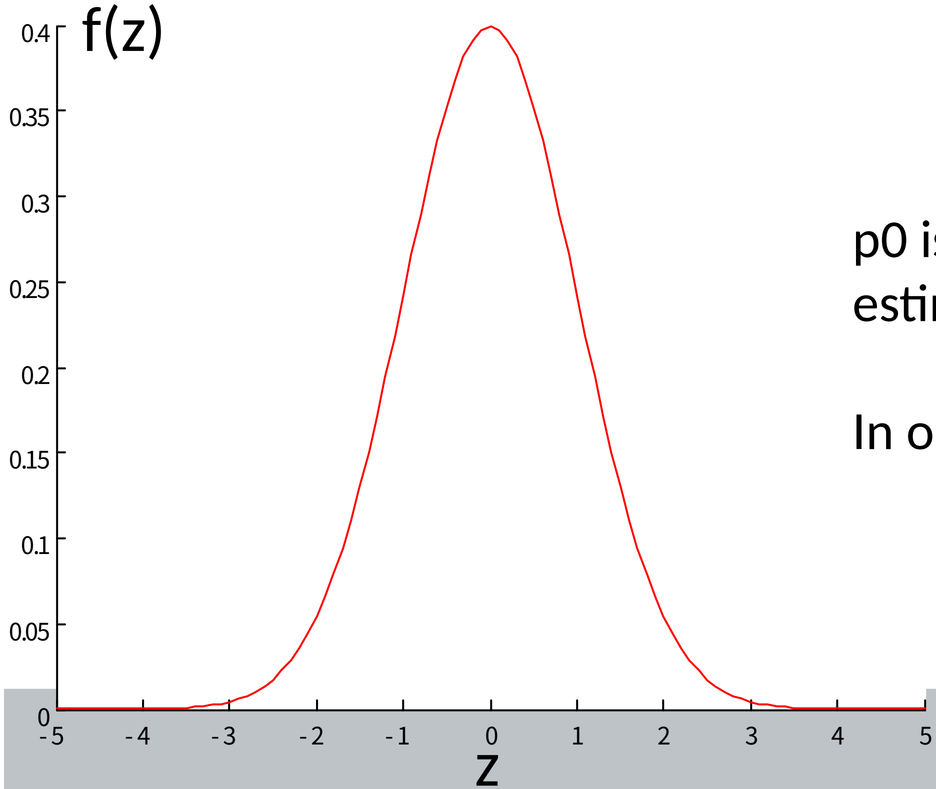
$$Z = \frac{\hat{p}}{\sqrt{p_0 q_0/n}} - \frac{p_0}{\sqrt{p_0 q_0/n}} = \frac{\hat{p} - p_0}{\sqrt{p_0 q_0/n}}$$

Compare to standard normal

We can standardize our estimated proportion and compare it to a value :

$$Z = \frac{\hat{p}}{\sqrt{p_0 q_0 / n}} - \frac{p_0}{\sqrt{p_0 q_0 / n}} = \frac{\hat{p} - p_0}{\sqrt{p_0 q_0 / n}}$$

and approximate the distribution of z with the standard normal distribution N(0,1).



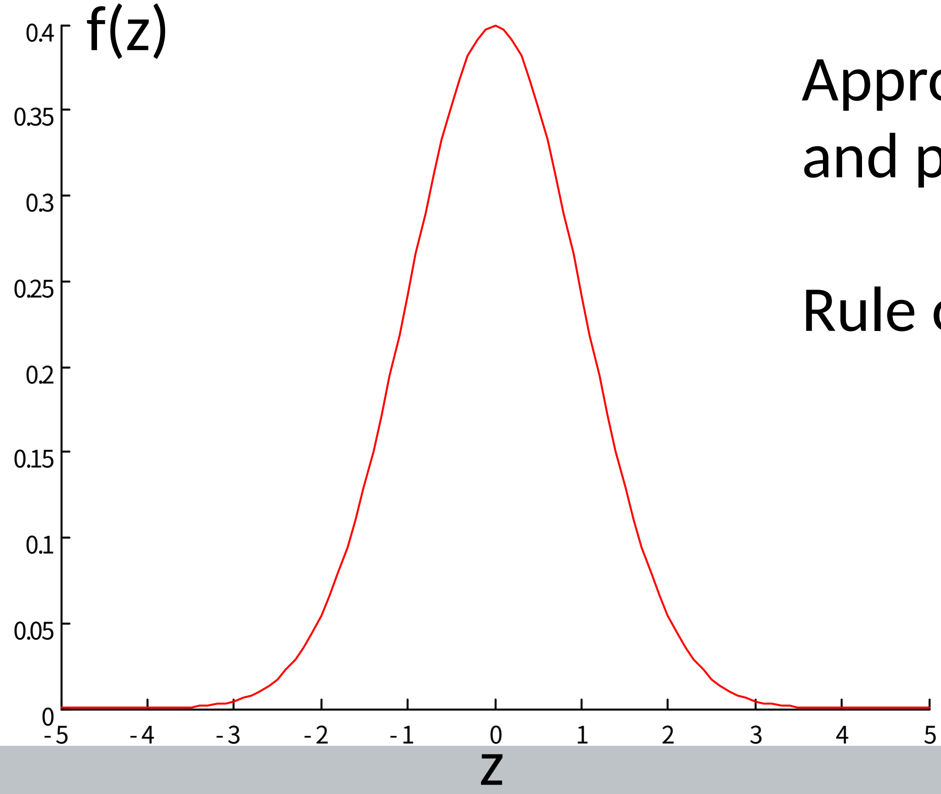
p0 is the value we want to compare our estimated proportion with.

In our case (with coins): $p0 = 0.5$

$q0 = 1 - p0 = 0.5$

Approximate with standard normal

We approximate the distribution of z with the standard normal distribution N(0,1). (Because remember we looked at distribution of X1-X2 before, and it was roughly normal?)

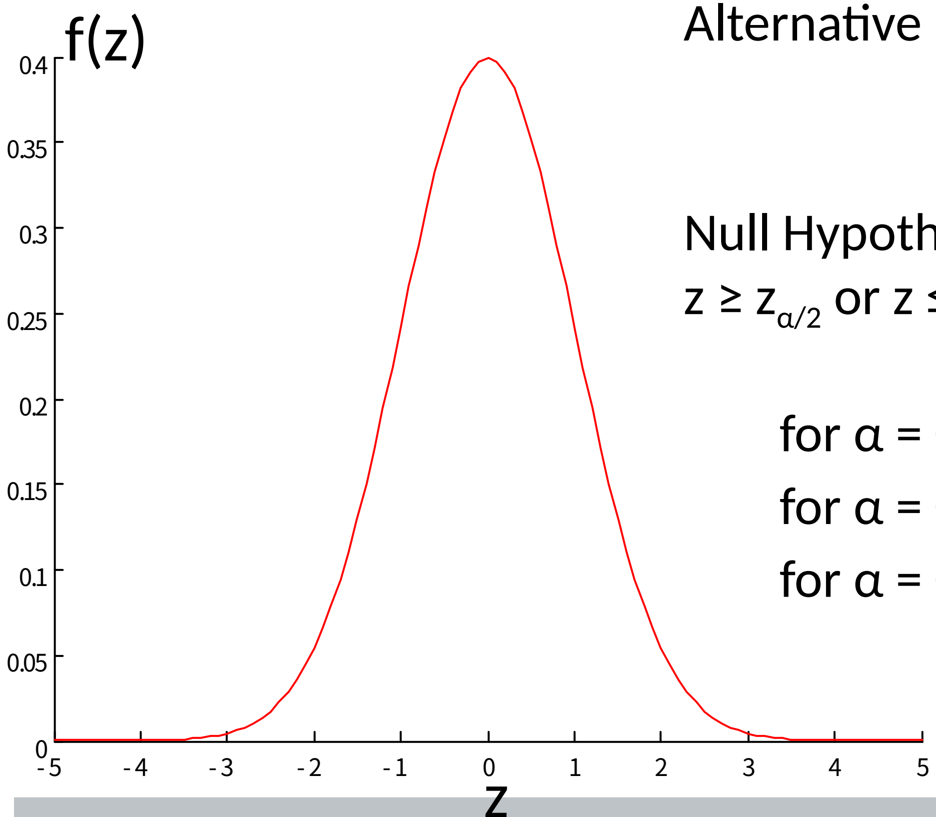


Approximation needs large n and p not too close to 0 or 1.

Rule of thumb:

$np \geq 5$ and $nq \geq 5$

Two-tailed test of one proportion



<u>Two-tailed test</u>:

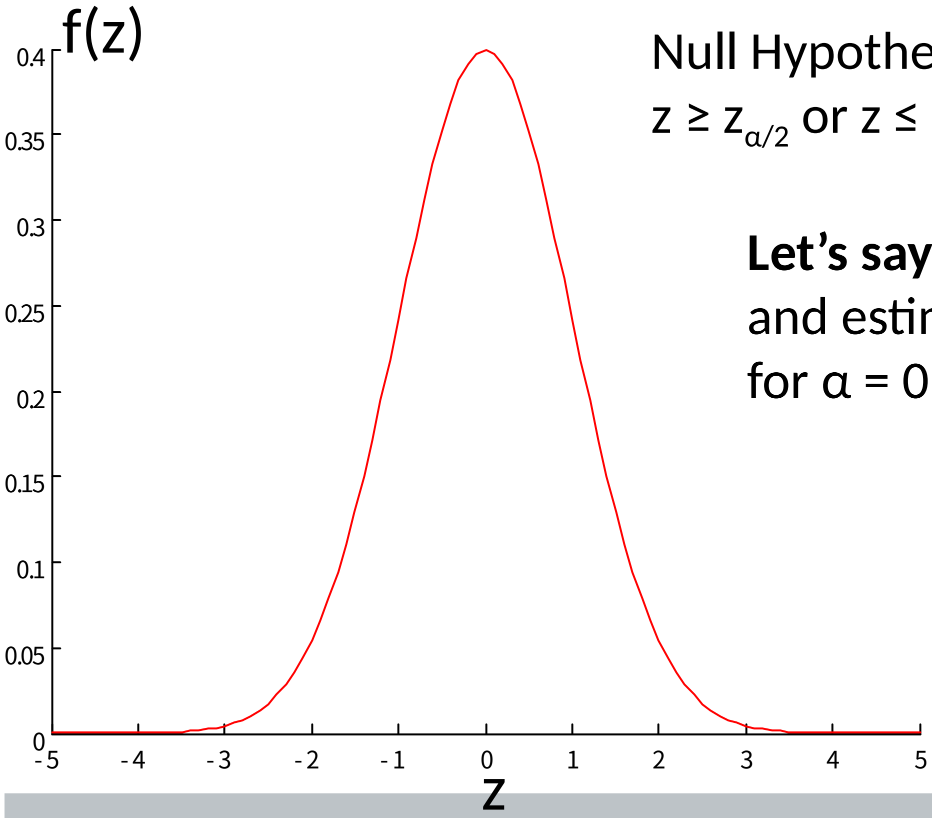
Null Hypothesis (H_0): $p = p0$

Alternative Hypothesis (H_a): $p \neq p0$

Null Hypothesis is rejected when
$z \geq z_{\alpha/2}$ or $z \leq -z_{\alpha/2}$

for $\alpha = 0.1$: $z_{\alpha/2} = 1.645$
for $\alpha = 0.05$: $z_{\alpha/2} = 1.960$
for $\alpha = 0.01$: $z_{\alpha/2} = 2.576$

Example: 30 coin flips, 21 heads



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Let's say we did 30 flips,

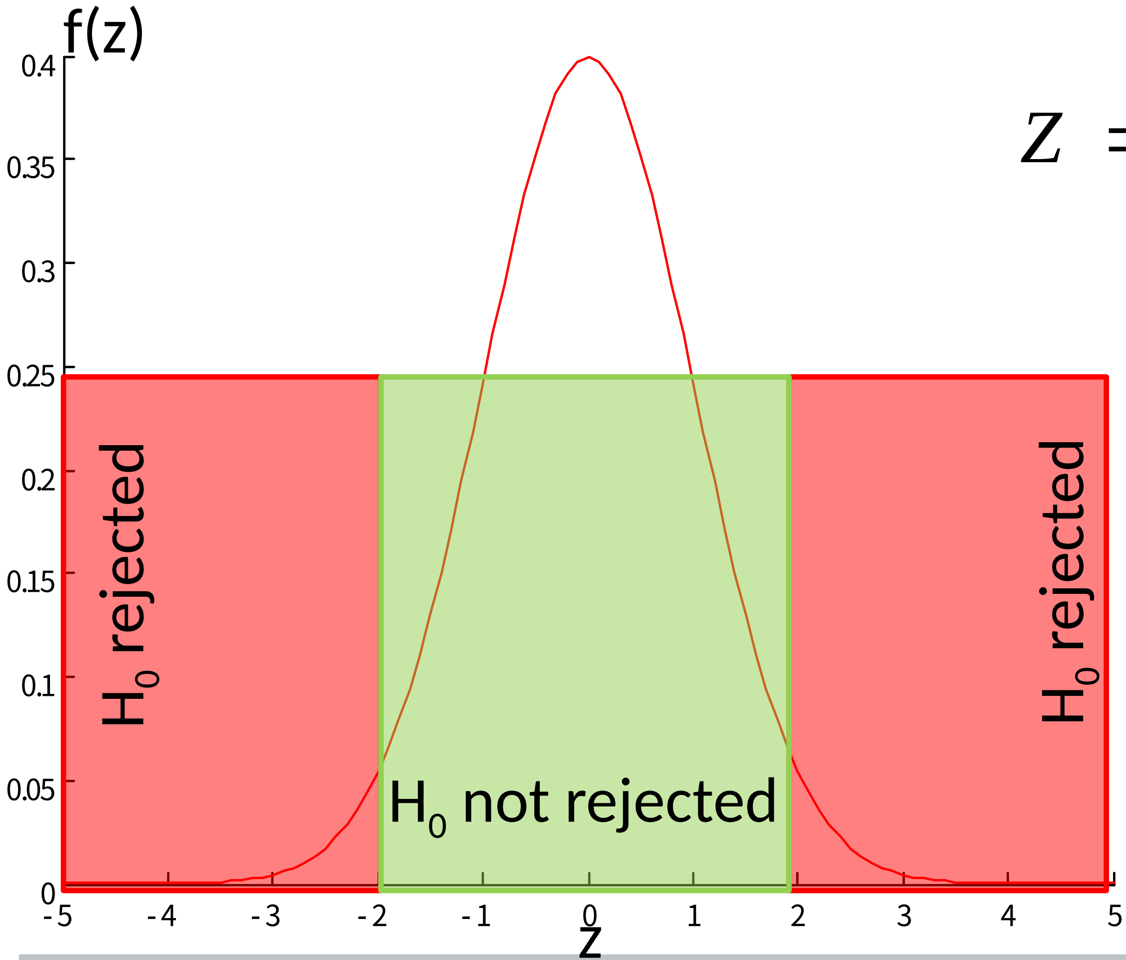
and estimate p hat = 0.7 (**21 Heads**)

for $\alpha = 0.05$: $z_{\alpha/2} = 1.960$

Example: 30 coin flips, 21 heads

We reject the null hypothesis that the coin (21 Heads in 30 flips) is unbiased, since $z \geq z_{\alpha/2}$, because $2.19 \geq 1.96$

$$Z = \frac{\hat{p} - p_0}{\sqrt{p_0 q_0 / n}} = 2.19$$



What about two proportions (difference)?

$$Z = \frac{\widehat{p}_1 - \widehat{p}_2 - \delta}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{m} + \frac{1}{n}\right)}}$$

(Delta is 0 for us – we just want to check if they're different)

$$\hat{p} = \frac{m}{m+n}\widehat{p}_1 + \frac{n}{m+n}\widehat{p}_2$$

$$\hat{q} = 1 - \hat{p}$$

m: sample1 size
n: sample2 size

Two-tailed test:

Null Hypothesis (H_0): $p_1 = p_2$

Alternative Hypothesis (H_a): $p_1 \neq p_2$

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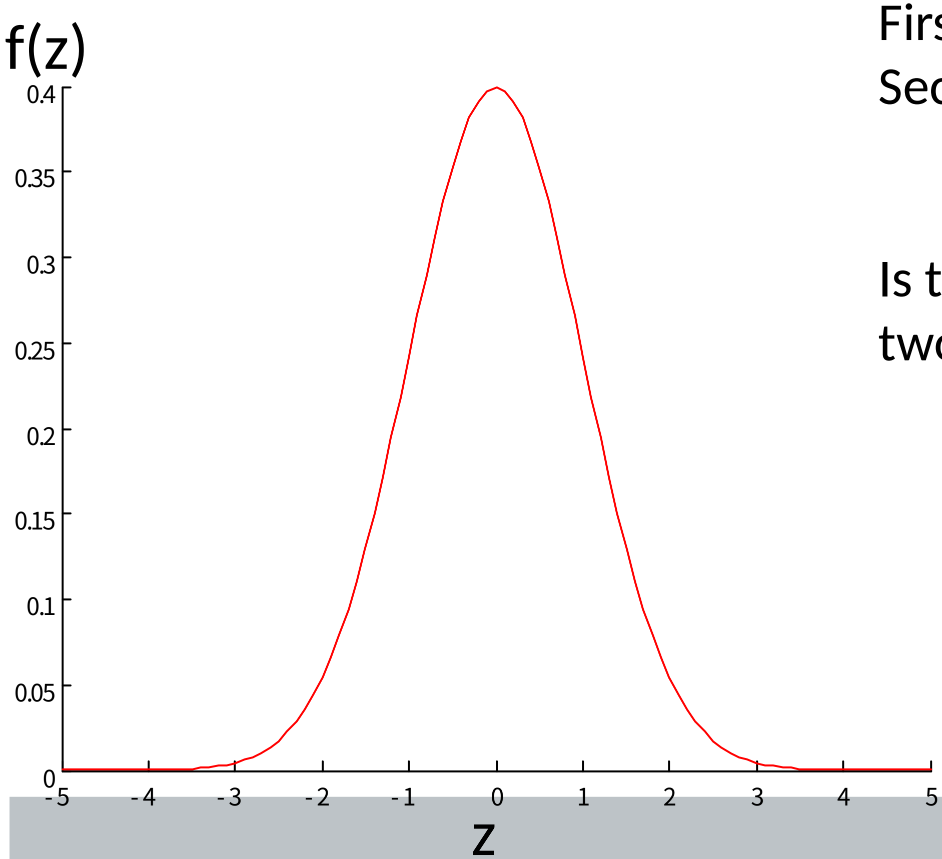
Example: difference 2 proportions



Example:

Two coins, flipped 20 times each

First coin: 15/20 Heads
Second coin: 9/20 Heads

Is there a difference between the two coins?

for $\alpha = 0.05$: $z_{\alpha/2} = 1.960$

Example: difference 2 proportions

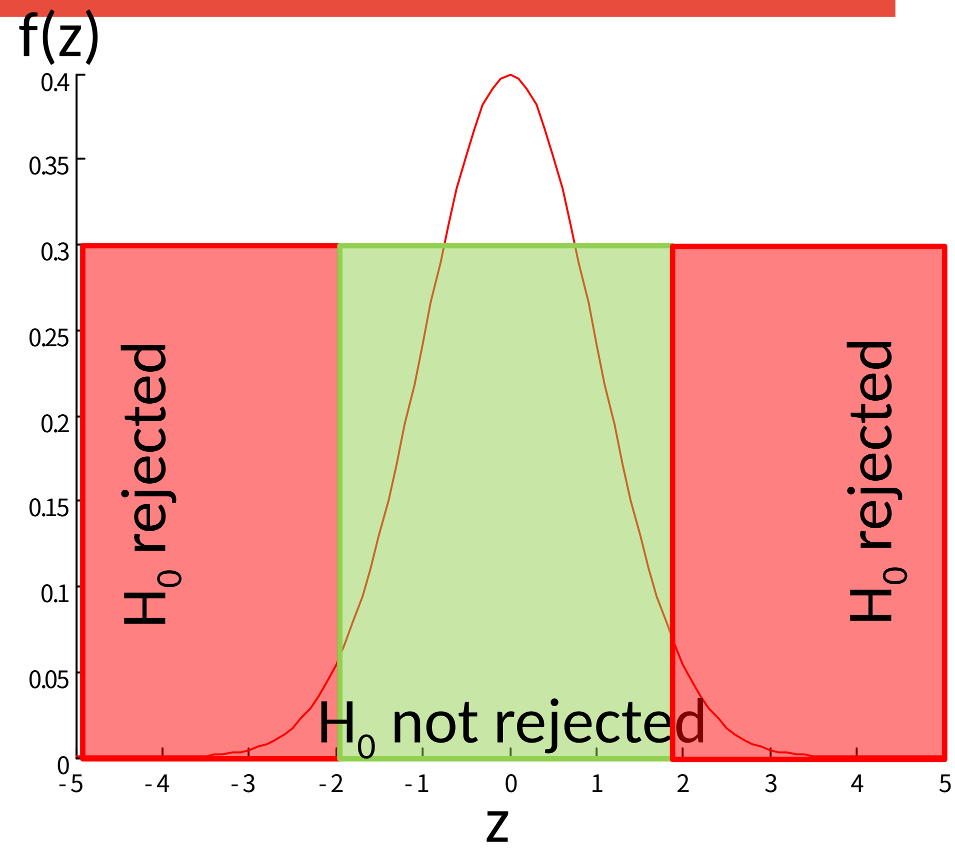
$$\hat{p}_1 = 15/20 \qquad \hat{p}_2 = 9/20$$

$$m = n = 20$$

$$\hat{p} = \frac{20}{20+20}\frac{15}{20} + \frac{20}{20+20}\frac{9}{20} = 0.6$$

$$\hat{q} = 1 - 0.6 = 0.4$$

$$Z = \frac{\hat{p}_1 - \hat{p}_2 - 0}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{m}+\frac{1}{n}\right)}} = 1.936$$

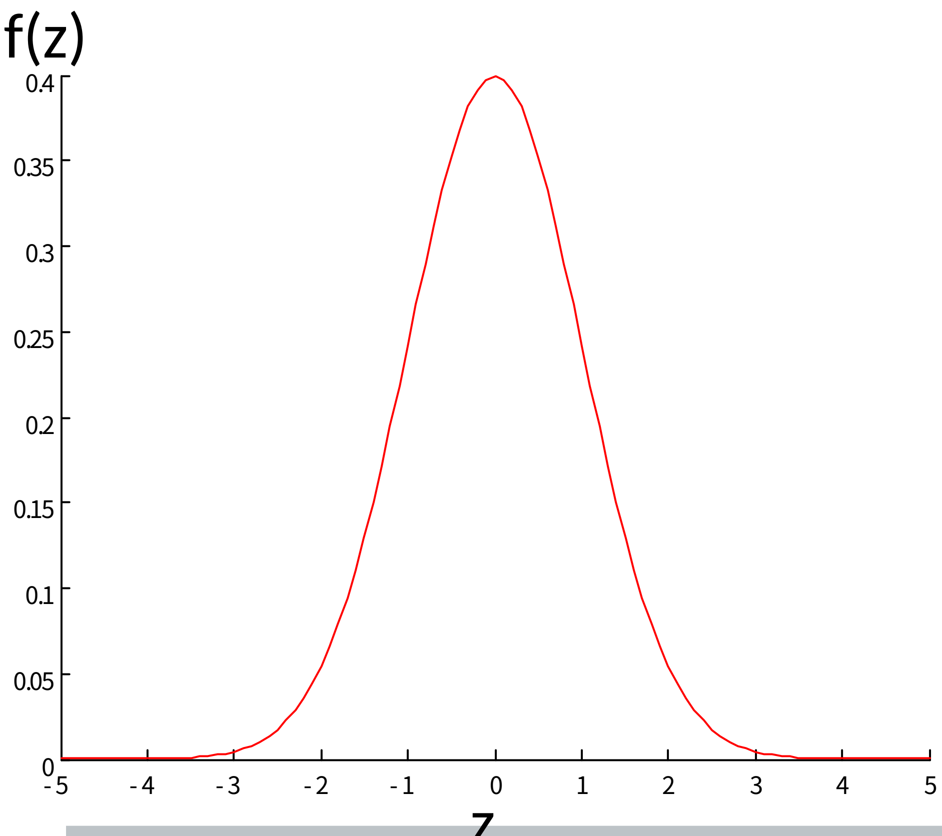


This z-value of 1.936 is smaller than the critical value 1.96 -> We keep the null hypothesis, there is no significant difference between the two coins.

Back to: Semmelweis, 1861 data

1st ward, Physicians: 9.92% (n=20042)

2nd ward, Midwifes: 3.88% (n=17791)



Two-tailed test:

Null Hypothesis (H_0): $p_1 = p_2$

Alternative Hypothesis (H_a): $p_1 \neq p_2$

Null Hypothesis is rejected when

$z \geq z_{\alpha/2}$ or $z \leq -z_{\alpha/2}$

for $\alpha = 0.01$: $z_{\alpha/2} = 2.576$

Semmelweis's data

$\widehat{p}_1 = 0.0992 \qquad \widehat{p}_2 = 0.0388$

$m = 20042 \qquad n = 17791$

$$\hat{p} = \frac{20042}{20042+17791}0.0992 + \frac{17791}{20042+17791}0.0388 = 0.0684$$

$$\hat{q} = 1 - 0.0684 = 0.9316$$

$$Z = \frac{\widehat{p}_1 - \widehat{p}_2 - 0}{\sqrt{\hat{p}\,\hat{q}\left(\frac{1}{m}+\frac{1}{n}\right)}} = 22.38$$

Semmelweis's data

$\widehat{p}_1 = 0.0992 \qquad \widehat{p}_2 = 0.0388$

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$$\hat{q} = 1 - 0.0684 = 0.9316$$

$$Z = \frac{\widehat{p}_1 - \widehat{p}_2 - 0}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{m} + \frac{1}{n}\right)}} = 22.38$$

That is huge.

22.38 is much larger than the critical value 2.576
→ We reject the null hypothesis

Semmelweis’s data

The data observed by Semmelweis from 1841-1846 suggests a significantly higher proportion of mothers’ death due to childbed fever on the 1st ward (9.92%) compared to the 2nd ward (3.88%), using a statistical test comparing two proportions ($z = 22.38$; $p<0.0001$).

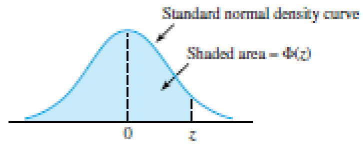
	1st ward: Physicians			2nd ward: Midwifes		
	births	dead	%	births	dead	%
1841	3036	237	7·7	2442	86	3·5
1842	3287	518	15·8	2659	202	7·5
1843	3060	274	8·9	2739	164	5·9
1844	3157	260	8·2	2956	68	2·3
1845	3492	241	6·8	3241	66	2·03
1846	4010	459	11·4	3754	105	2·7
Summa	20042	1989	9·92	17791	691	3·38

Semmelweis, 1861

Z-tables

Table A.3 Standard Normal Curve Areas

$\Phi(z) = P(Z \leq z)$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0038
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3482
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

(continued)

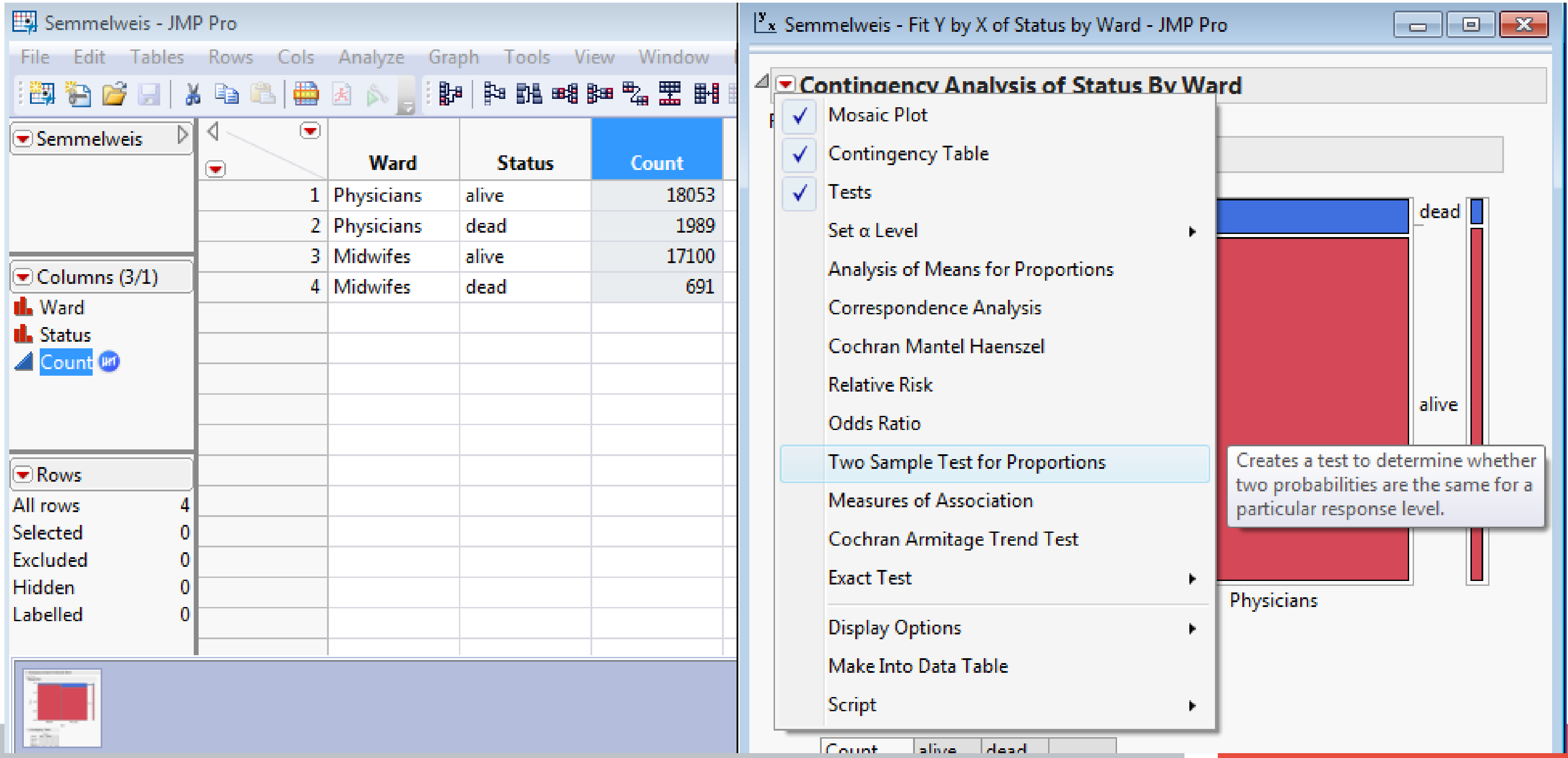
Table A.3 Standard Normal Curve Areas (cont.) $\Phi(z) = P(Z \leq z)$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Critical values can be looked up in these pre-calculated tables that show the areas under the curve of the standard normal function.

Z-test in JMP

We can use the “Fit Y by X” dialog box and then select the red triangle and choose “Two sample test for Proportions”.



Z-test in JMP

This will give us additional information, including the confidence interval and the p-value of the z-test (JMP: adjusted Wald test).

Two Sample Test for Proportions

Description	Proportion Difference	Lower 95%	Upper 95%
P(dead|Midwifes)-P(dead|Physicians)	-0.0604	-0.06541	-0.05537

Adjusted Wald Test	Prob
P(dead|Midwifes)-P(dead|Physicians) ≥ 0	1.0000
P(dead|Midwifes)-P(dead|Physicians) ≤ 0	<.0001*
P(dead|Midwifes)-P(dead|Physicians) = 0	<.0001*

Response Status category of interest

- ○ alive
- ◉ dead

We have to define what is our category of interest: in this case alive or dead.

Confidence Intervals: Problems

We used risk ratio to measure *effect size*.

Confidence Intervals are not so clear-cut:

I mentioned that our confidence intervals for relative risk rely on *Central Limit Theorem*

→ The distribution of outcomes caused by adding together many independent random variables (doesn't matter their distribution!) is ***normal*** (with appropriate transformation)

→ You need to have enough *N*.

→ The more "weird" the distribution is, the more *N* you need to make it normal.

→ *This confidence interval is not* ***exact.***

There are many ways to estimate confidence interval for *difference* of proportion, but none of them are good – so we will not cover it.

Confidence interval (one proportion)

If your observed proportion is p hat:

Where z is the cutoff for your alpha
This is sometimes called “Wald interval”

$$SD\ of\ binomial: \sqrt{pq/n}$$

$$CI = \hat{p} \pm \left(z \times SE\right)$$

$$\left(Estimated\right) SE = \sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{n}}$$

There are lots of issues with this
1) You really need to correct from discrete binomial
to continuous Normal (“continuity correction”),
especially for smaller N

Confidence interval for difference between two proportions

There are so many (bad) ways to do this. We can use the simplest method by chi-square:

$$SE = \sqrt{\frac{\hat{p}_1\left(1-\hat{p}_1\right)}{m} + \frac{\hat{p}_2\left(1-\hat{p}_2\right)}{n}}$$

$$CI = \hat{p}_1 - \hat{p}_2 \pm \left(z \times SE\right)$$

We *really* need n and m > 30
and np1 > 5 and np2 > 5