Introductory Statistics
9: Sample Size

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https://youtu.be/pcyfJUFJMQs

Lecture Video at above link

Summary

(Another) short lecture:

1) Sample size calculation

→ What size N of samples do we need to achieve a certain α with a certain confidence?

Coins again…

How large should my sample be?

Let’s say we want to test whether a coin is manipulated….

How many times do we have to flip a coin to be able to reject the null hypothesis $p = 0.5$?

Null Hypothesis (H_0): $p = 0.5$

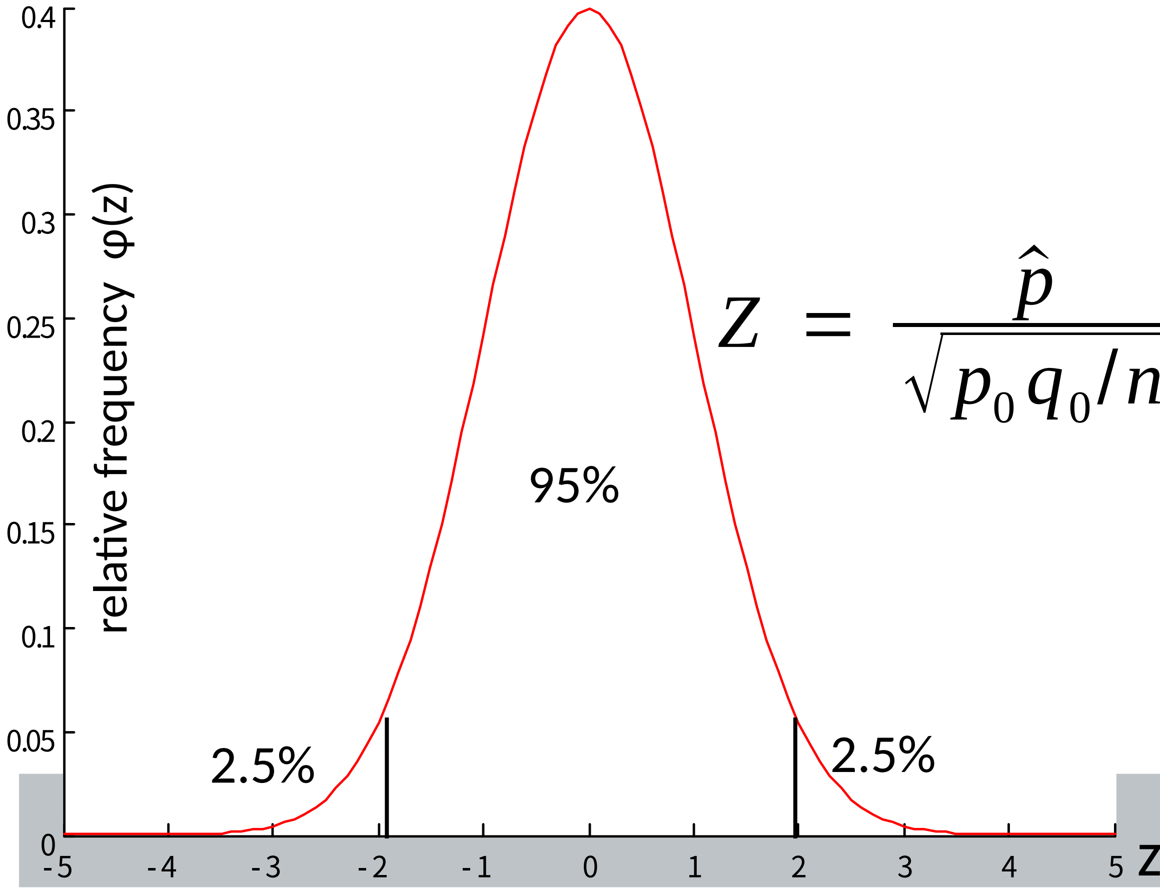
Alternative Hypothesis (H_a): $p \neq 0.5$

Sample size calculation

Null Hypothesis (H_0): $p = 0.5 = p_0$

Alternative Hypothesis (H_a): $p \neq 0.5$

Type I error (α): rejecting H_0 even though it is true



here: $\alpha = 0.05$

$z_{crit} = \pm 1.96$

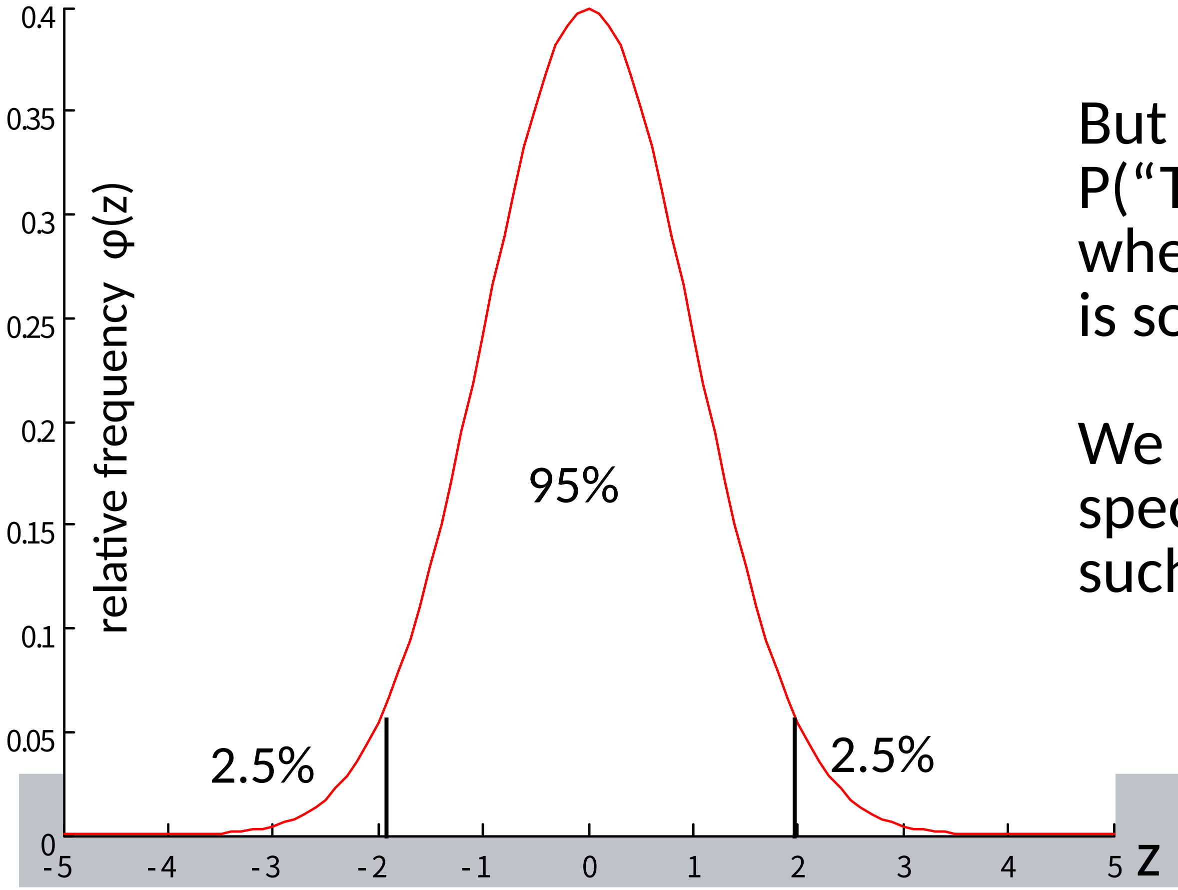
$$Z = \frac{\hat{p}}{\sqrt{p_0 q_0 / n}} - \frac{p_0}{\sqrt{p_0 q_0 / n}} = \frac{\hat{p} - p_0}{\sqrt{p_0 q_0 / n}}$$

We need to specify type II error rate

Null Hypothesis (H_0): $p = 0.5 = p_0$

Alternative Hypothesis (H_a): $p \neq 0.5$

Type II error (β): keeping H_0 even though it is not true



But how can we calculate P("Type II error") = β when our alternative hypothesis is so unspecific?

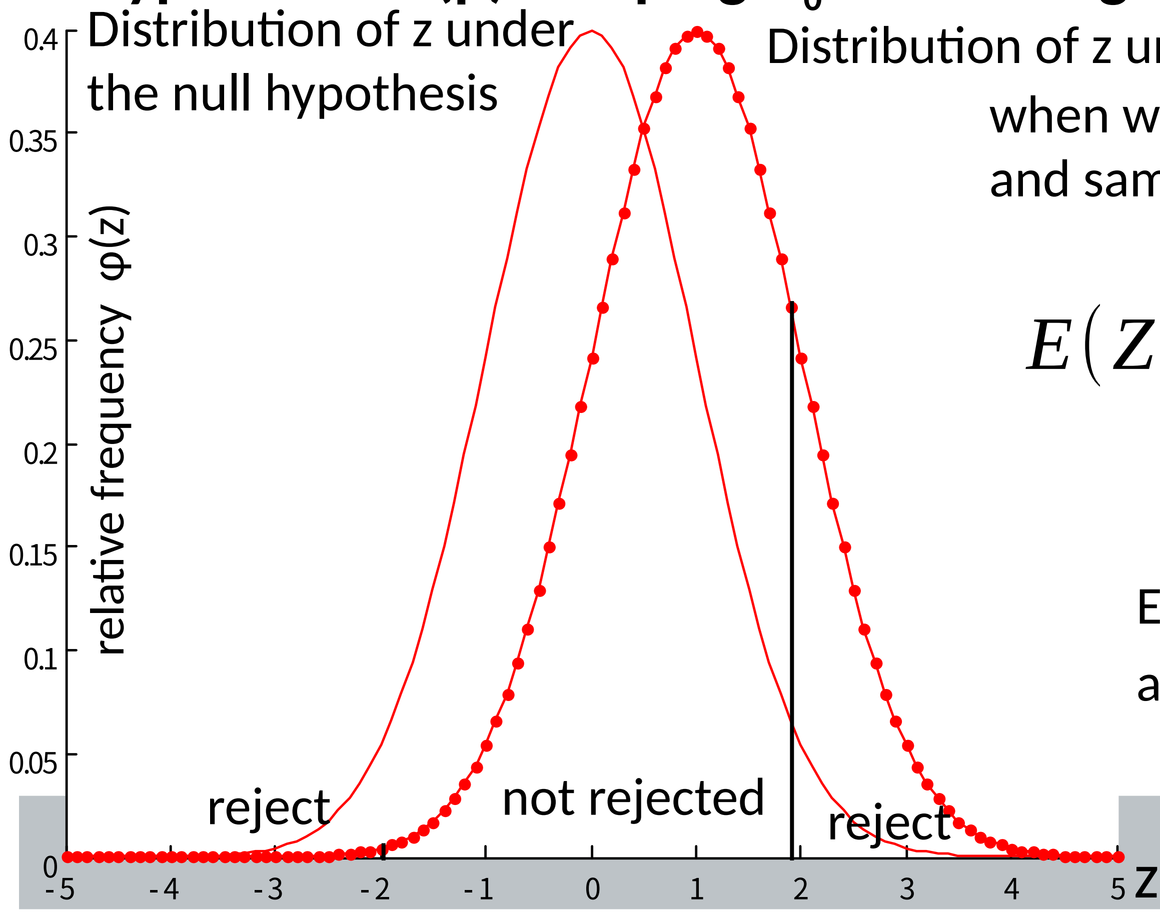
We cannot → we need a more specific alternative hypothesis, such as: $p = 0.55$

Type II error (beta)

Null Hypothesis (H_0): $p = 0.5 = p_0$

Alternative Hypothesis (H_a): $p = p_a = 0.55$

Type II error (β): keeping H_0 even though it is not true



Distribution of z under the alternative hypothesis when we assume p = 0.55 and sample size n = 100

$$E(Z_a) = \frac{p_a - p_0}{\sqrt{p_0 q_0 / n}}$$

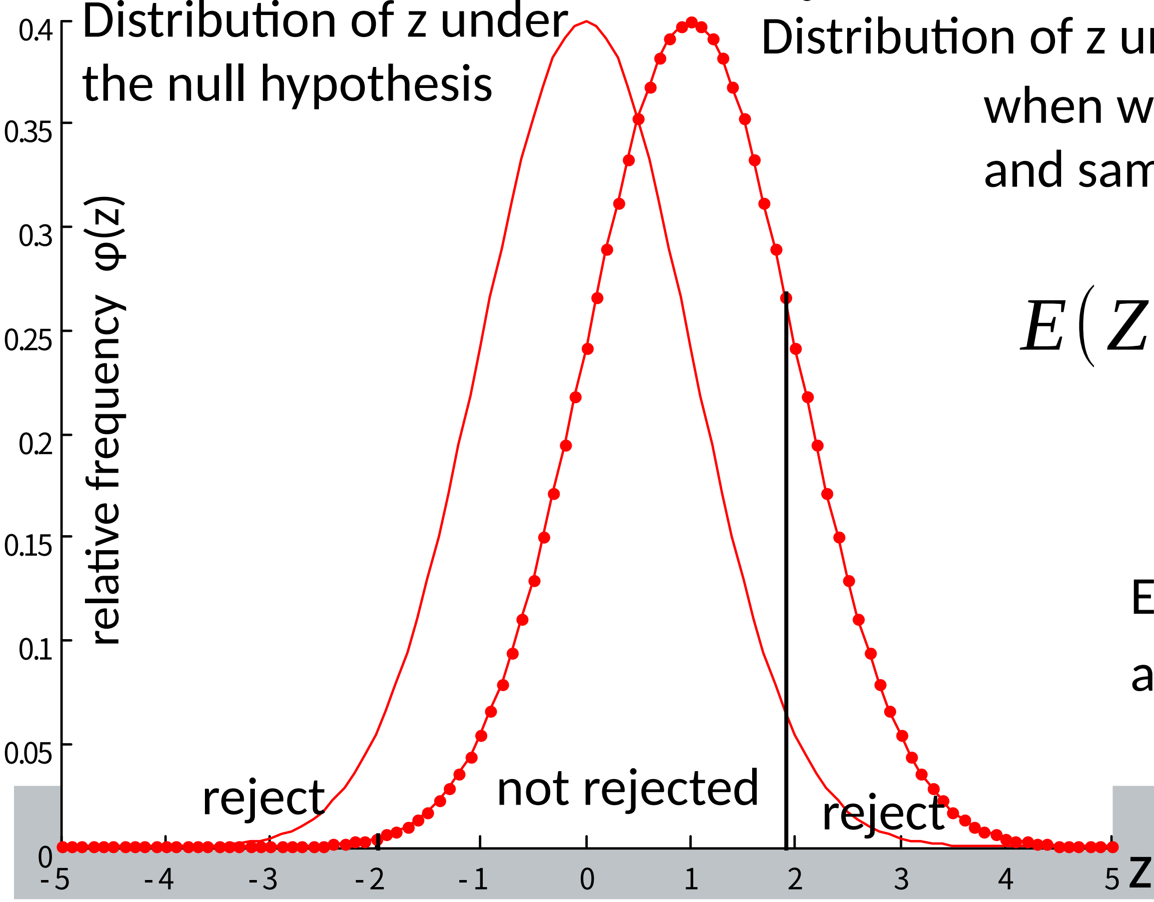
$E(Z_a)$: expected value for z if the alternative hypothesis is true.

Computing type II error

Null Hypothesis (H_0): $p = 0.5 = p_0$

Alternative Hypothesis (H_a): $p = p_a = 0.55$

Type II error (β): keeping H_0 even though it is not true



Distribution of z under the alternative hypothesis when we assume p = 0.55 and sample size n = 100

$$E(Z_a) = \frac{0.55 - 0.50}{\sqrt{0.5 \times 0.5 / 100}} = 1$$

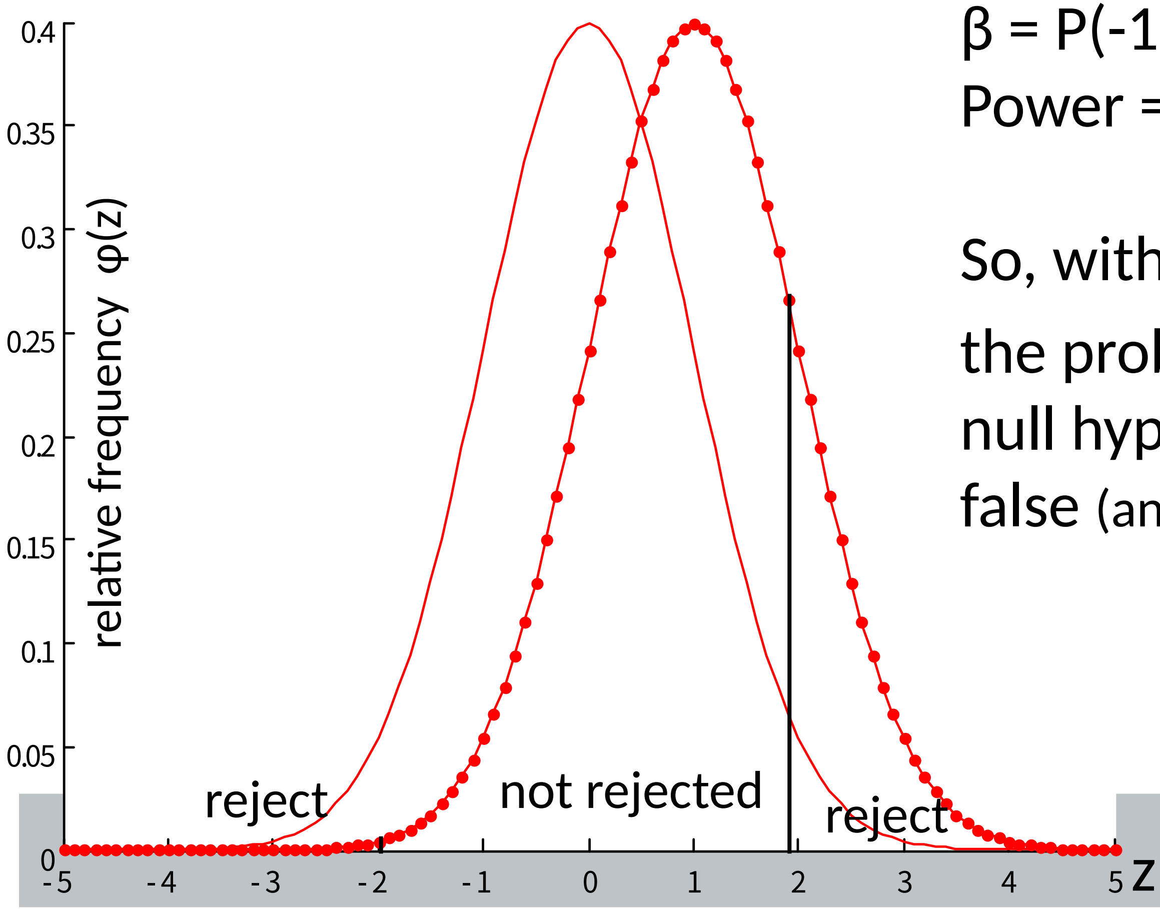
$E(Z_a)$: expected value for z if the alternative hypothesis is true.

Computing type II error

Null Hypothesis (H_0): $p = 0.5 = p_0$

Alternative Hypothesis (H_a): $p = p_a = 0.55$

We can compute the area under the normal curve of the z-distribution assuming the alternative hypothesis:



$\beta = P(-1.96 < z < 1.96) = 0.83$

Power $= 1 - \beta = 0.17$

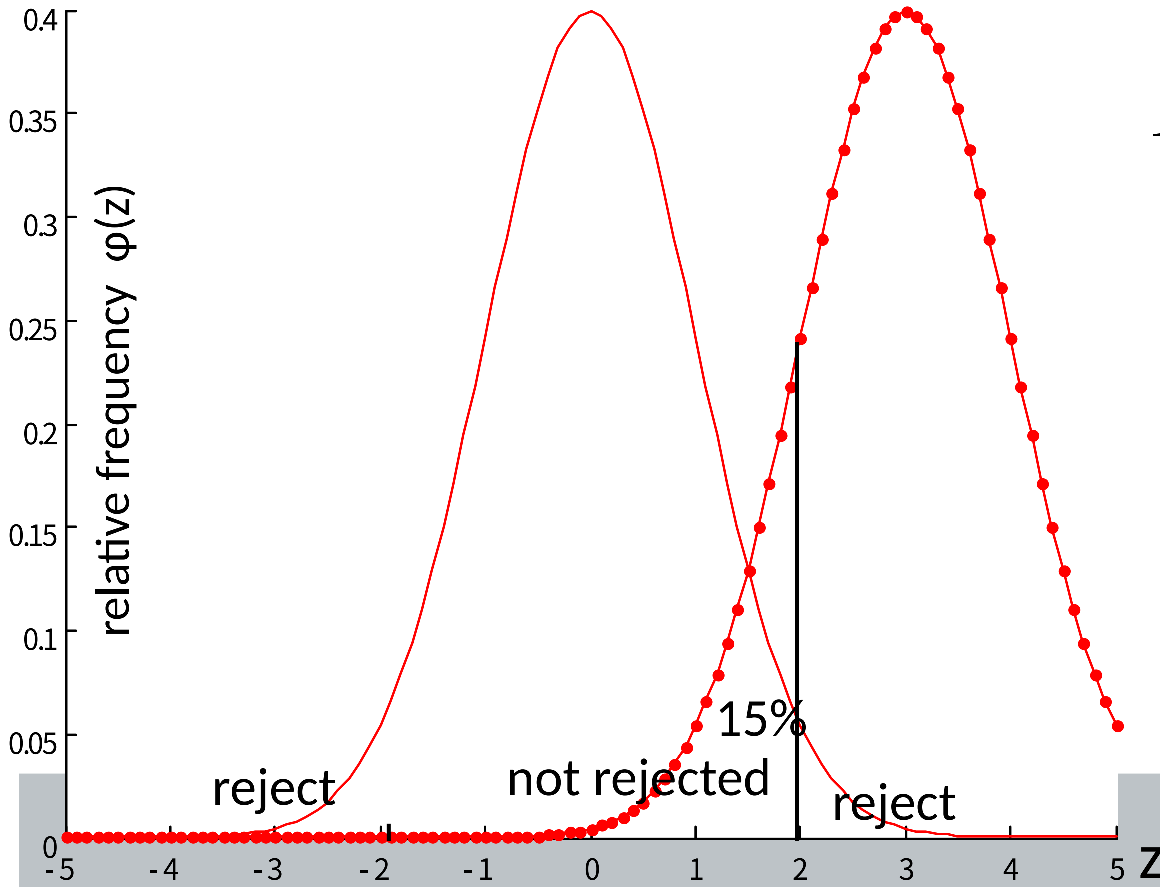
So, with $p_a = 0.55$ and n=100, the probability not to reject the null hypothesis even though it is false (and alternative is true) is 0.83.

Computing type II error

Null Hypothesis (H_0): p = 0.5 = p_0

Alternative Hypothesis (H_a): p = p_a = 0.55

Let's increase the sample size to 900:



$$E(Z_a) = \frac{p_a - p_0}{\sqrt{p_0 q_0 / n}}$$

$\beta = 0.15$

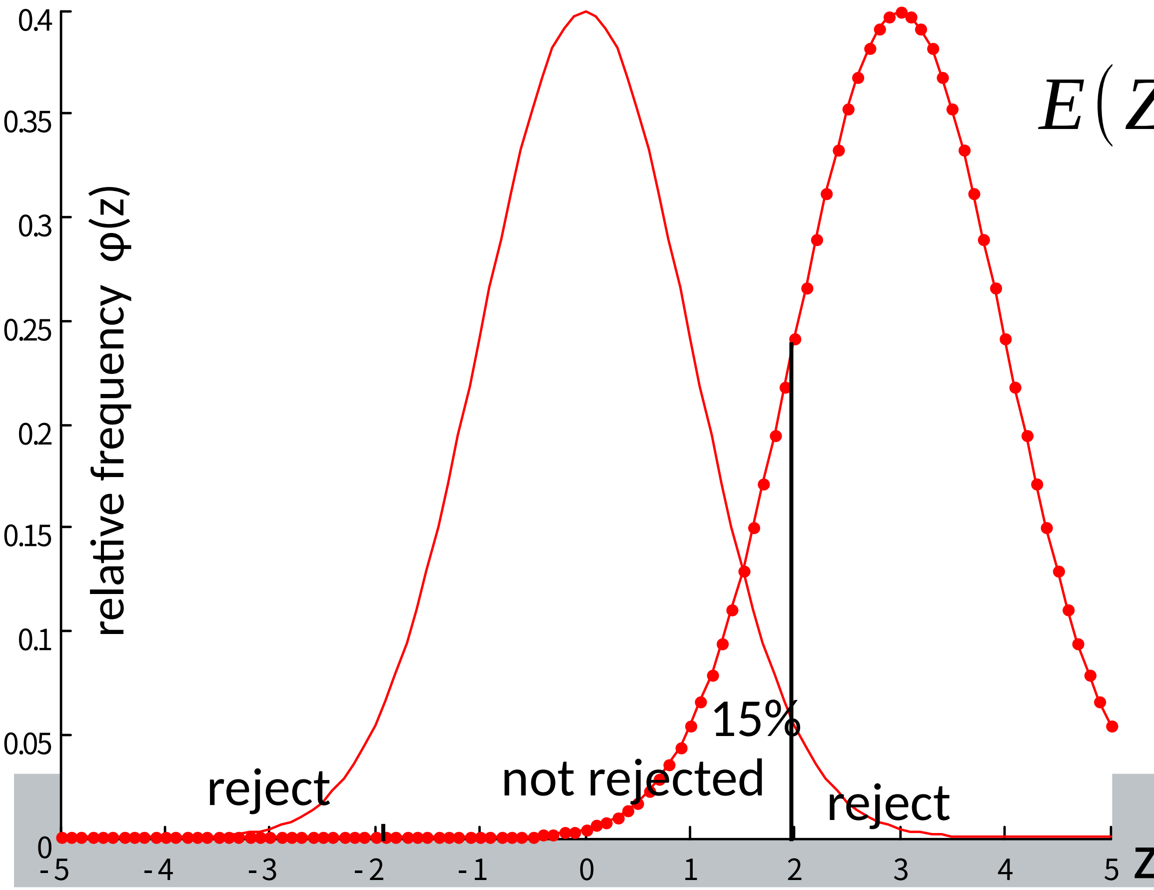
Power =1- β=0.85

Computing type II error

Null Hypothesis (H_0): $p = 0.5 = p_0$

Alternative Hypothesis (H_a): $p = p_a = 0.55$

Let's increase the sample size to 900:



$$E(Z_a) = \frac{0.55 - 0.50}{\sqrt{0.5 \times 0.5 / 900}} = 3$$

$\beta = 0.15$

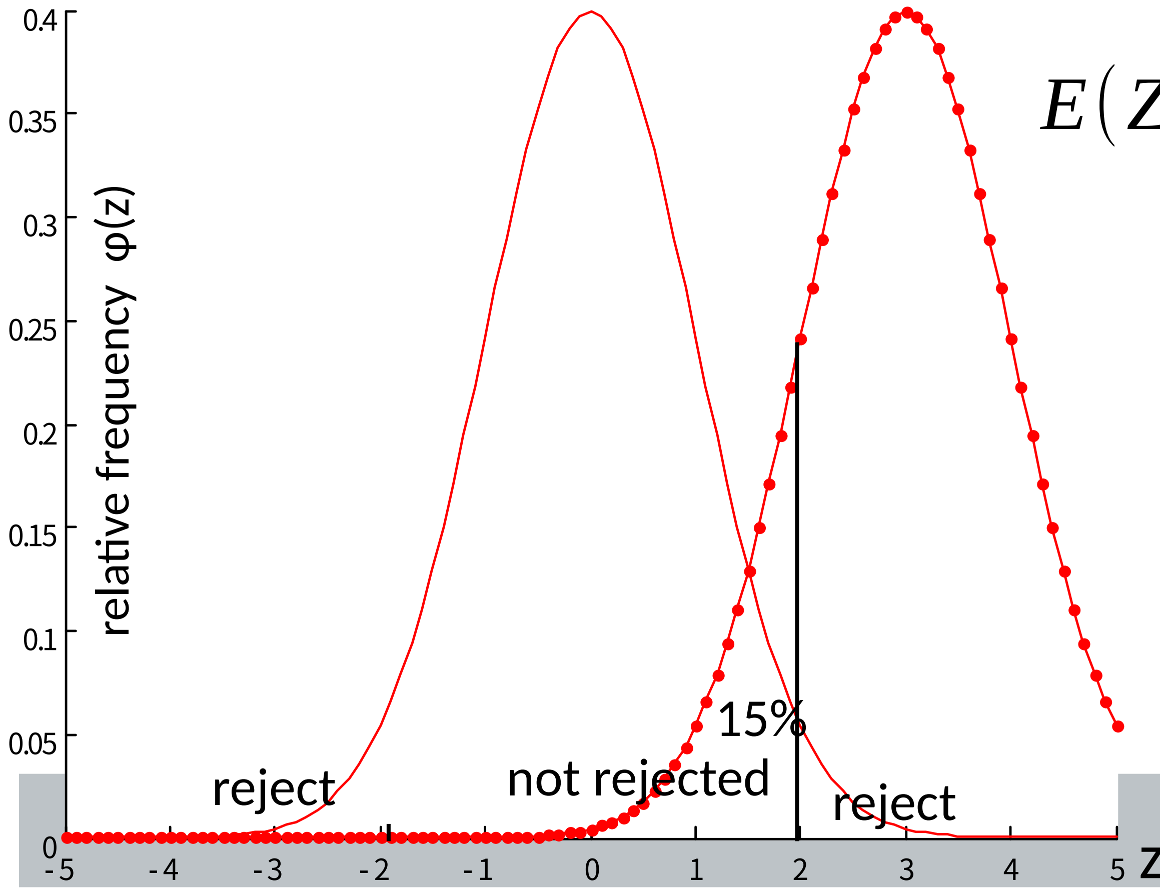
Power $= 1 - \beta = 0.85$

Computing type II error

Null Hypothesis (H_0): $p = 0.5 = p_0$

Alternative Hypothesis (H_a): $p = p_a = 0.65$

Let's go back to sample size 100 but increase p_a to 0.65



$$E(Z_a) = \frac{0.65 - 0.50}{\sqrt{0.5 \times 0.5 / 100}} = 3$$

$\beta = 0.15$

Power $= 1 - \beta = 0.85$

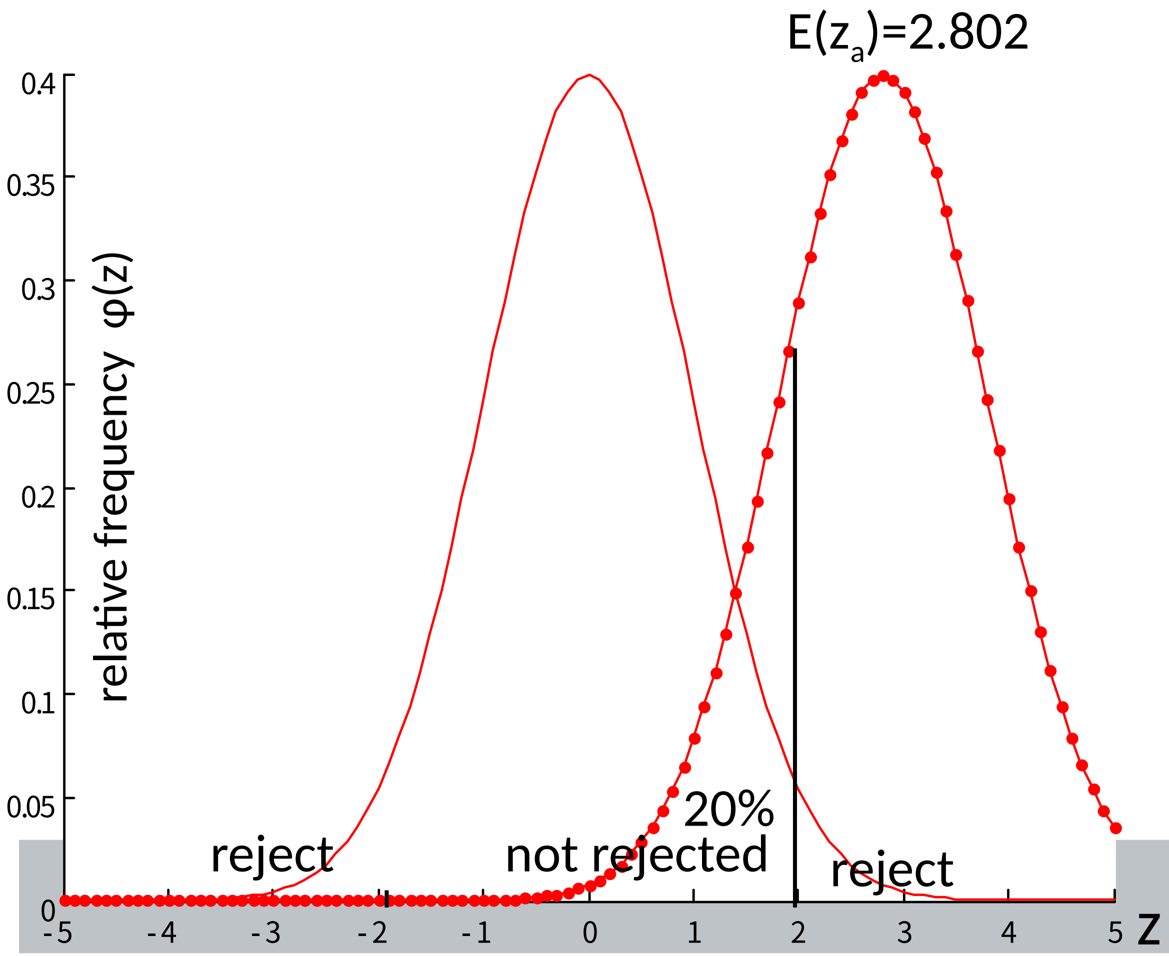
Going the other direction

Null Hypothesis (H_0): $p = 0.5$

Alternative Hypothesis (H_a): $p \neq 0.5$

Let's say, we want a power = 0.8, β = 0.2

And we want to be able to detect $P_a = 0.6$



Going the other direction

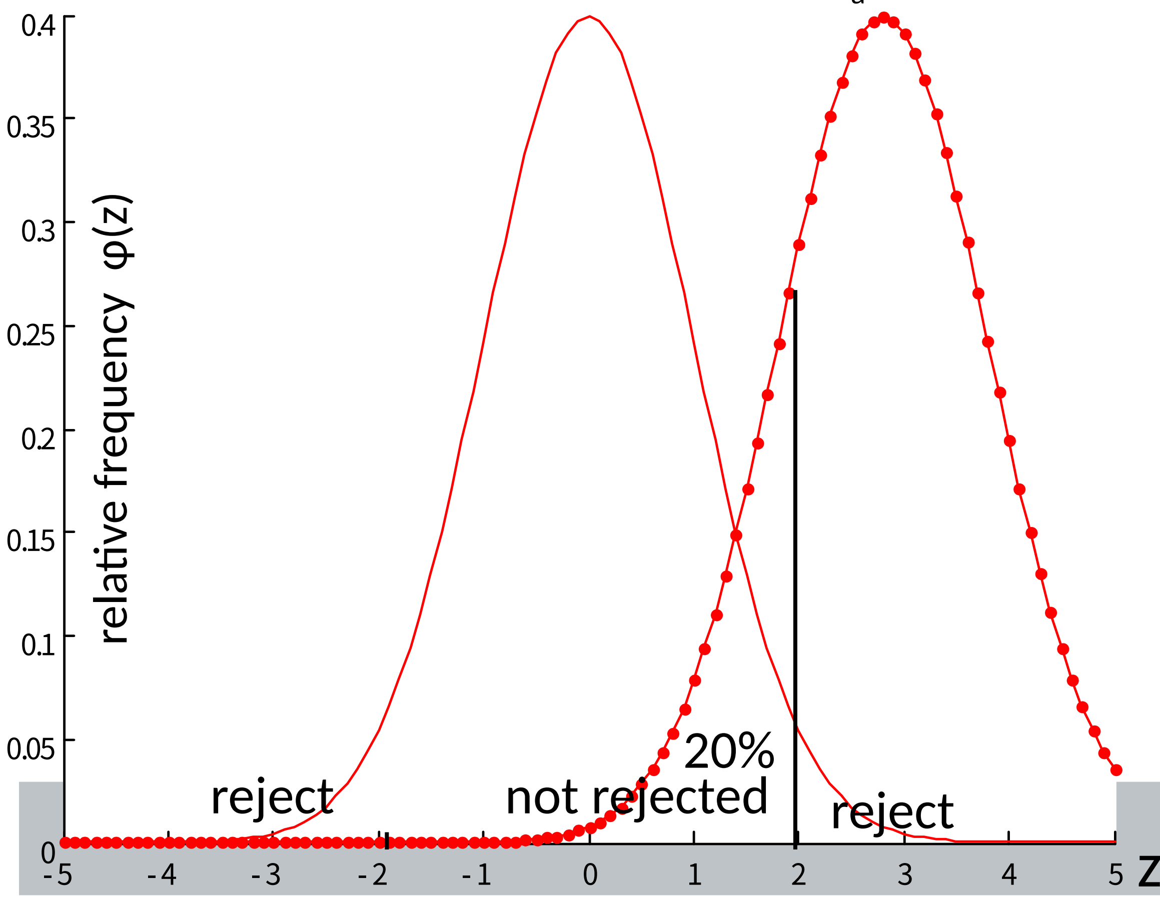
Null Hypothesis (H_0): $p = 0.5$

Alternative Hypothesis (H_a): $p \neq 0.5$

Let's say, we want a power = 0.8, β = 0.2

And we want to be able to detect $P_a = 0.6$

$E(z_a)$=2.802 → depends only on α and β



now we need to assume a typical p_a and find the corresponding *n*.

for α=0.01 and β=0.2: $E(z_a)$=3.418

for α=0.05 and β=0.1: $E(z_a)$=3.242

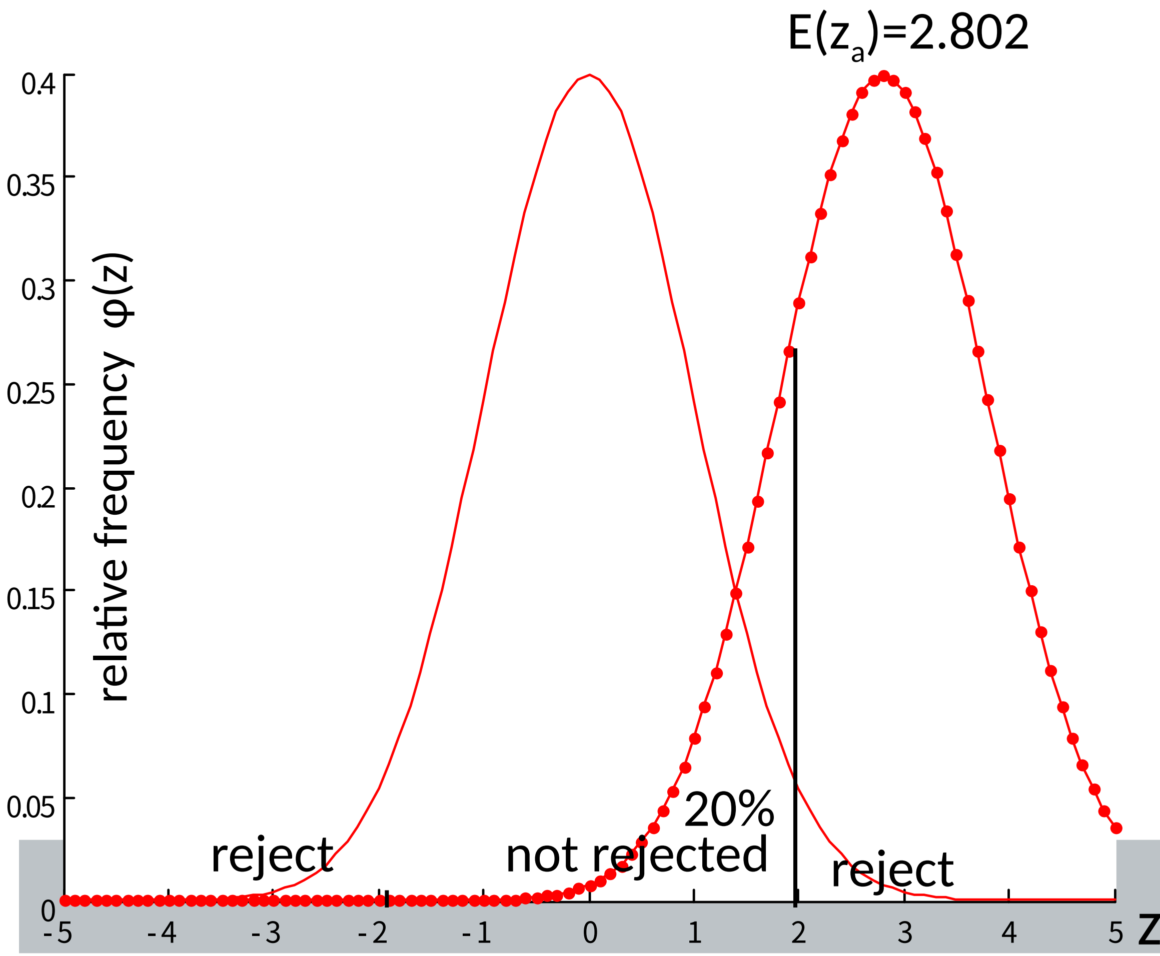
Going the other direction

Null Hypothesis (H_0): $p = 0.5$

Alternative Hypothesis (H_a): $p \neq 0.5$

Let's say, we want a power = 0.8, β = 0.2

And we want to be able to detect $P_a = 0.6$



$$E(Z_a) = \frac{p_a - p_0}{\sqrt{p_0 q_0 / n}}$$

$$2.802 = \frac{0.6 - 0.5}{\sqrt{0.5 \times 0.5 / n}}$$

→ Solve for *n*

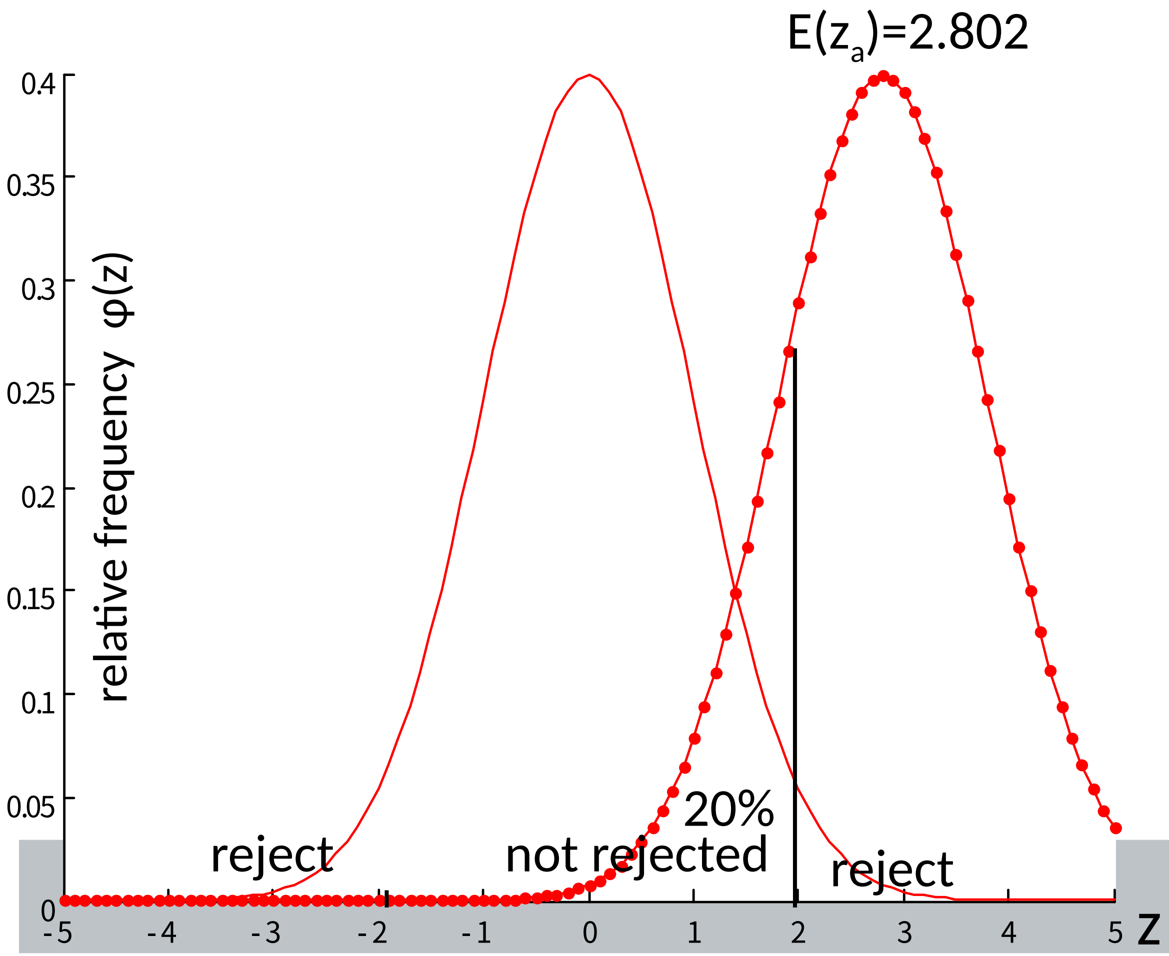
Going the other direction

Null Hypothesis (H_0): $p = 0.5$

Alternative Hypothesis (H_a): $p \neq 0.5$

Let's say, we want a power = 0.8, β = 0.2

And we want to be able to detect $P_a = 0.6$



$$E(Z_a) = \frac{p_a - p_0}{\sqrt{p_0 q_0 / n}}$$

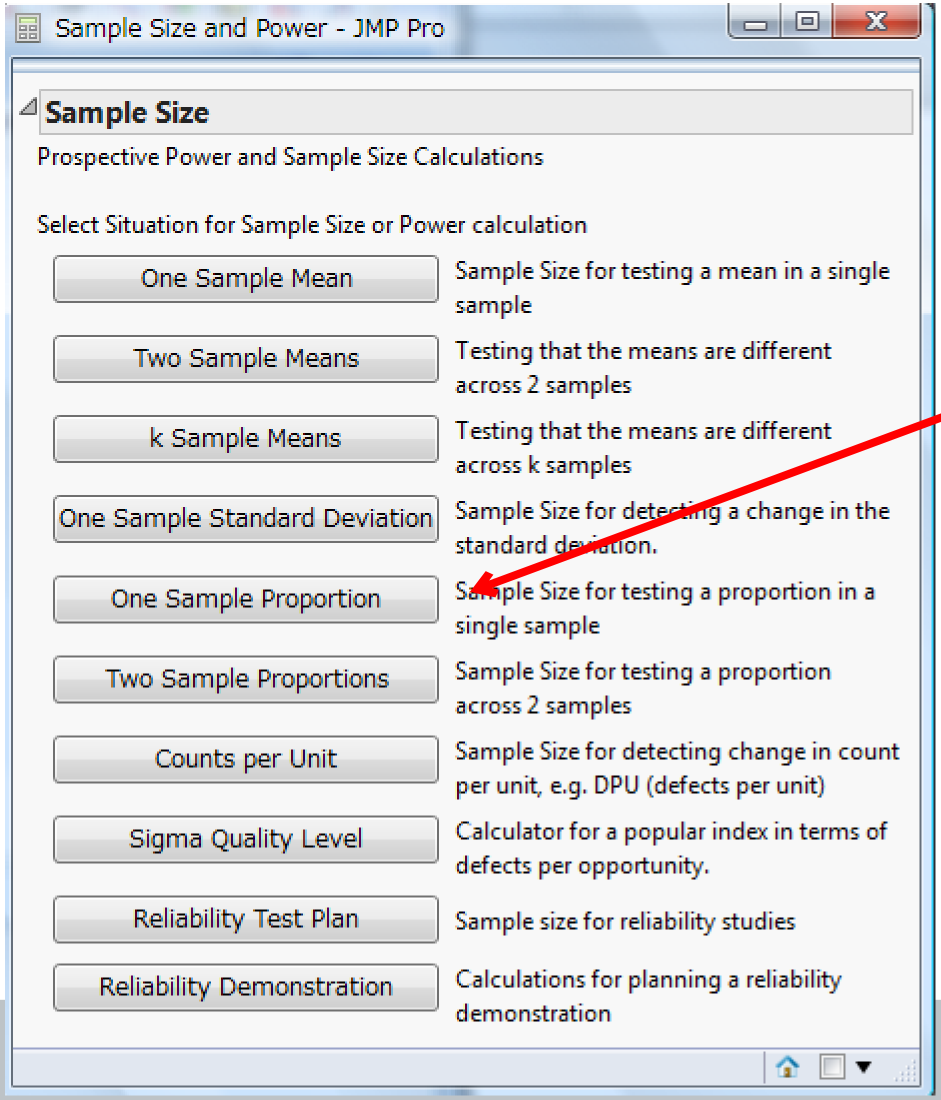
$$\sqrt{(n)} = z \frac{\sqrt{p_0 q_0}}{p_a - p_0}$$

$$n = z^2 \frac{p_0 q_0}{(p_a - p_0)^2}$$

$$n = 196$$

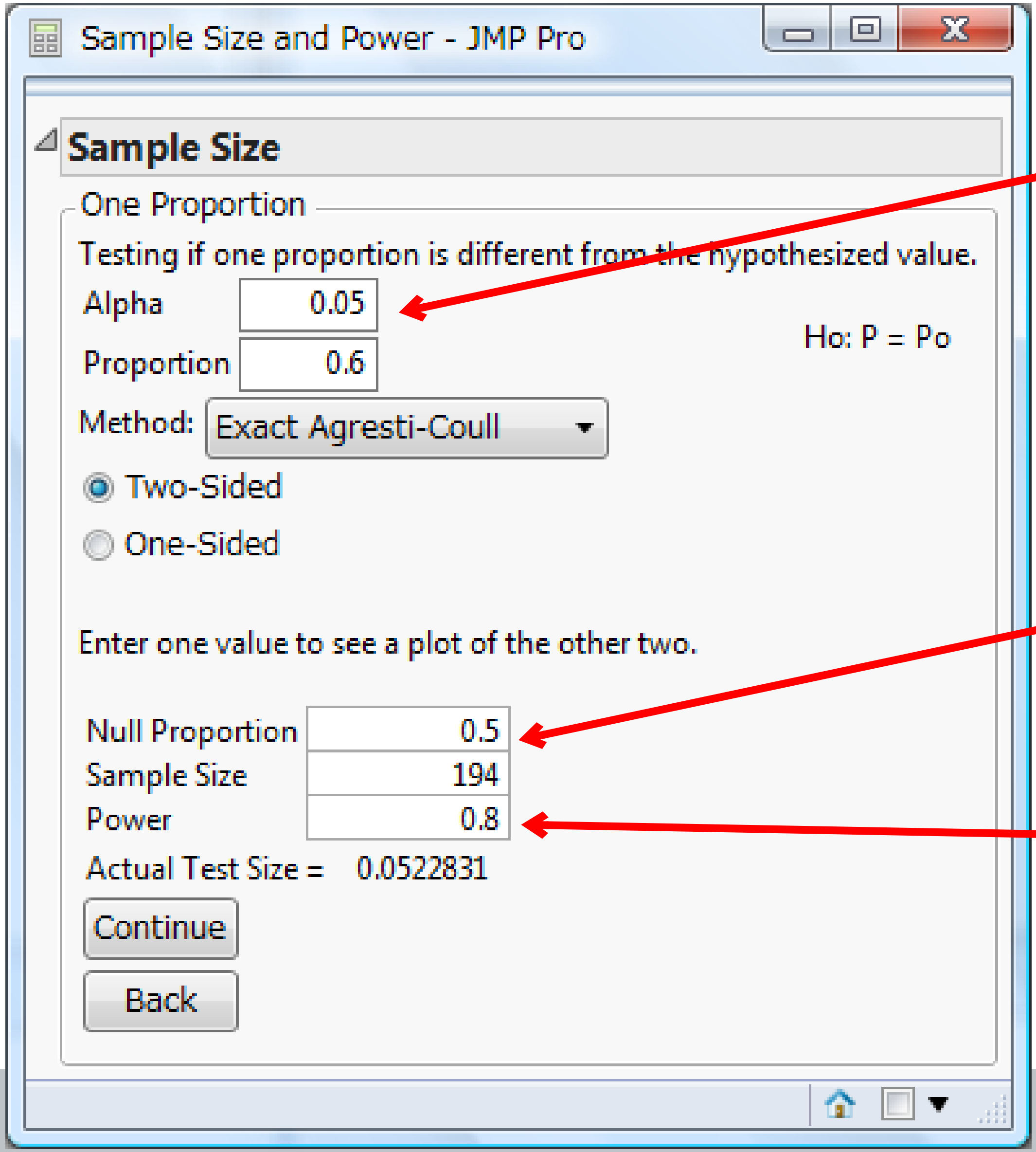
In JMP

DOE(Design of experiments)-> Sample Size and Power



For a single sample

In JMP



α=0.05 Type I Error

Proportion: What we think the alternative could be.

p_0

Power =1-β

What about *two* proportions…?

Let's say Dr. Semmelweis wants to test his hypothesis that washing hands will reduce childbed fever.


Ignaz Philipp Semmelweis (1818–65).

So, he wants to split the physicians' ward into a treatment group and a placebo group.

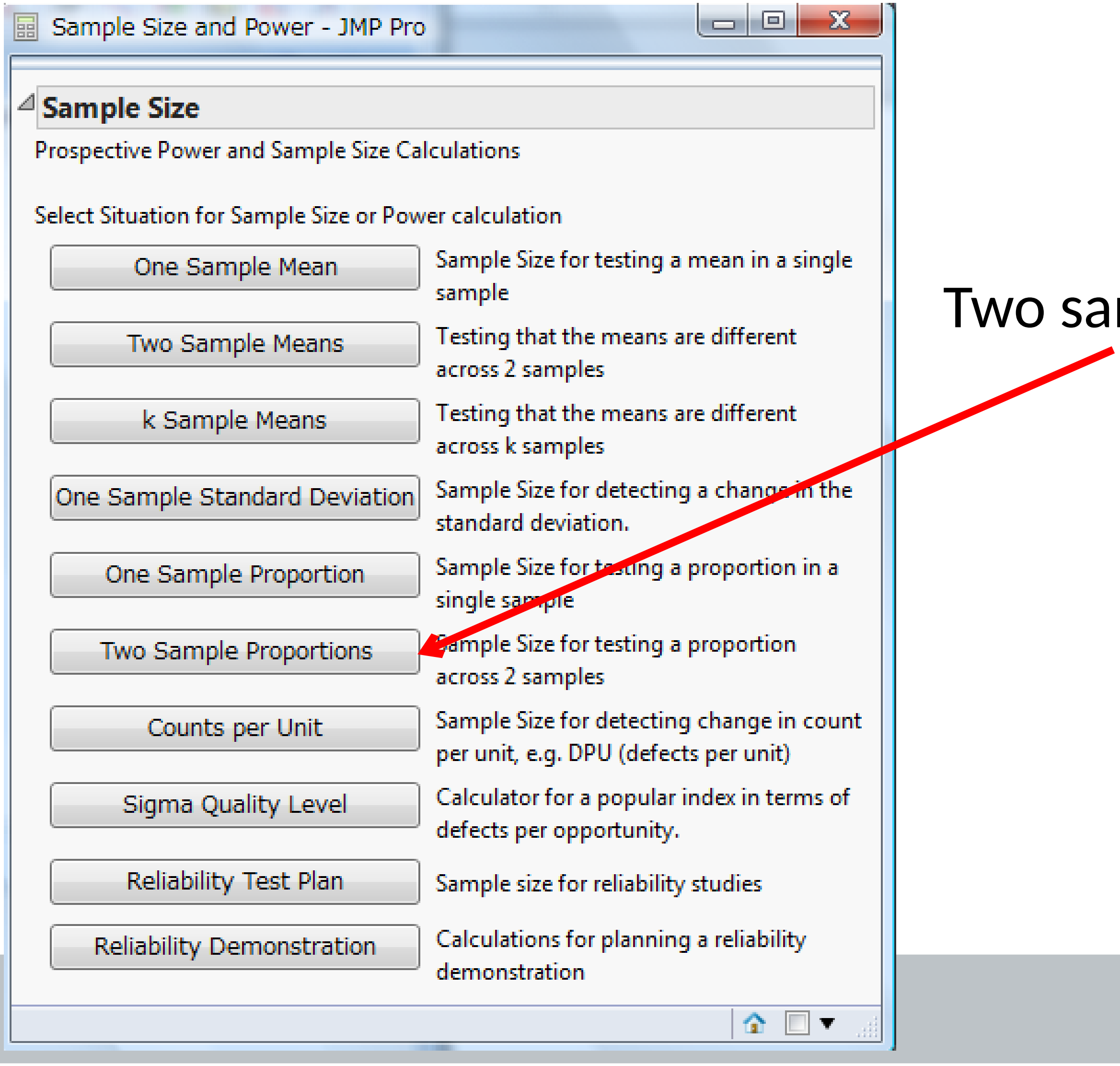
p_0 = 0.1; we want to be able to detect a decrease to 0.05

$p_{Placebo}$ = 0.1 p_{Treat} = 0.05

α = 0.05, β = 0.2 (power = 0.8)

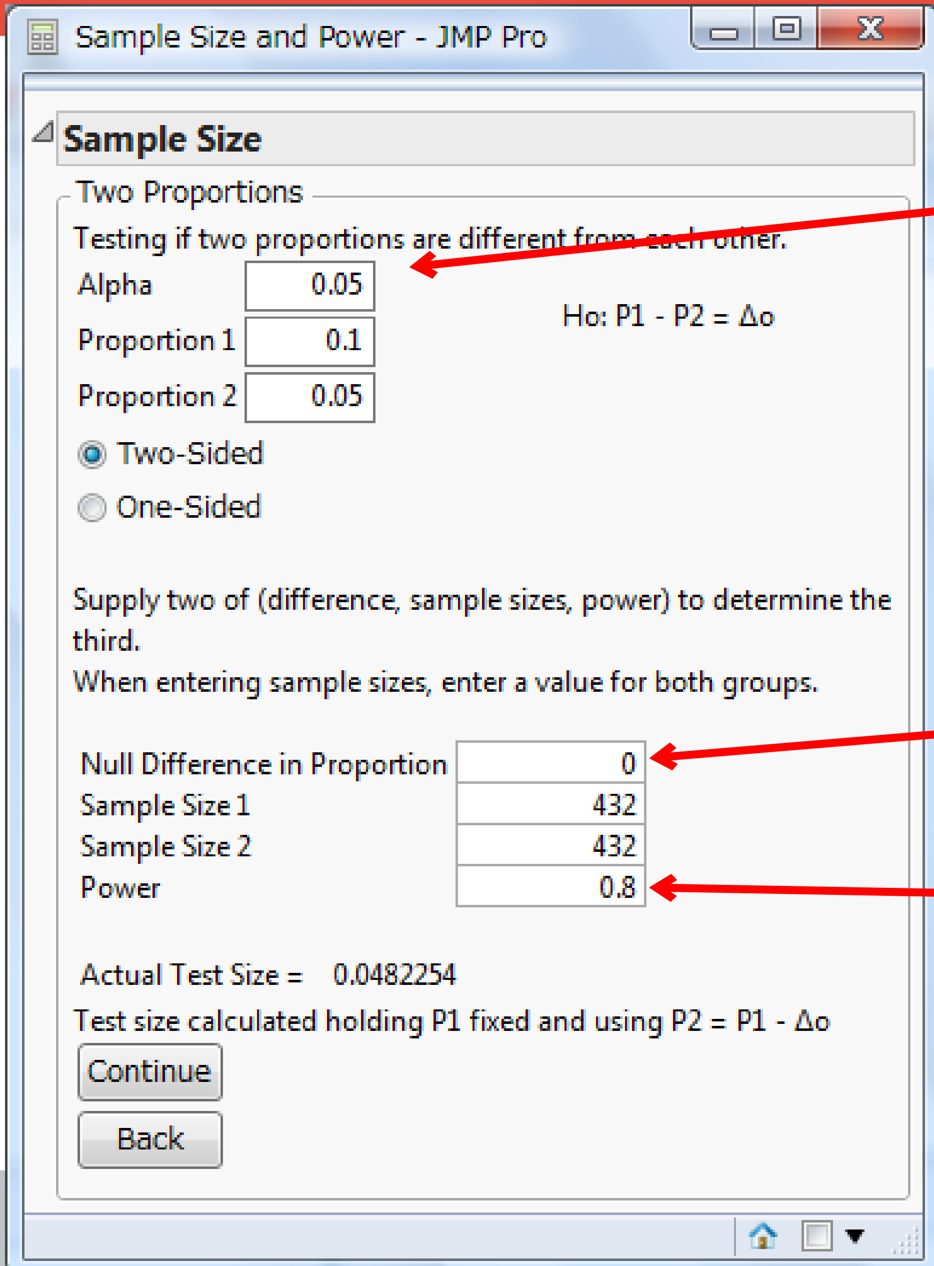
Two proportions in JMP

DOE-> Sample Size and Power



Two samples

Two proportions in JMP



α=0.05 Type I Error

Proportions: What we think the proportions p could be.

Difference under null hypothesis

Power =1-β

n=m=432

Summary: Sample Size Calculation

1) Choose the desired α and β (1-Power).

2) Choose the minimum difference of proportions you want to detect with the given α and β.

3) Calculate n:

for one proportion:

$$n = z^2 \frac{p_0 q_0}{(p_a - p_0)^2}$$

for difference of two proportions:

$$n = z^2 \frac{p_{a1} q_{a1} + p_{a2} q_{a2}}{(p_{a1} - q_{a2})^2}$$

for α=0.01 and β=0.1: z=3.858
for α=0.01 and β=0.2: z=3.418
for α=0.05 and β=0.1: z=3.242
for α=0.05 and β=0.2: z=2.802